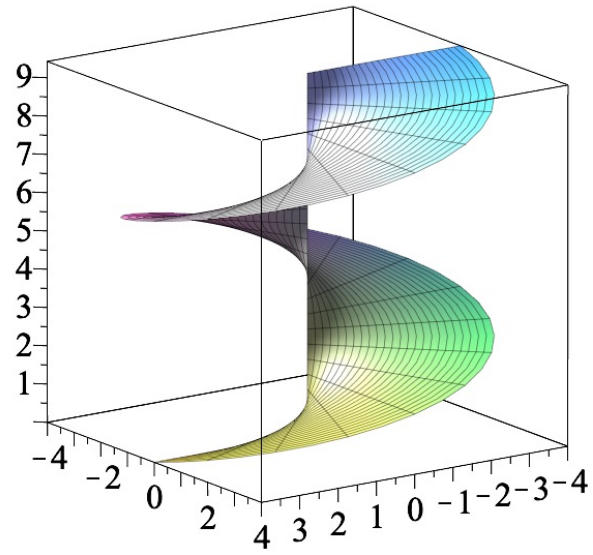
MATH 223: Multivariable Calculus



Class 8:Wednesday
September 27, 2023

- Notes on Assignment 7
- Assignment 8
- Unified Treatment Of Tangent Lines and Planes
- Parametrized Surfaces in *MATLAB* [Handouts Folder]

Announcements

Exam 1: Next Wednesday, 7 PM - No Time Limit

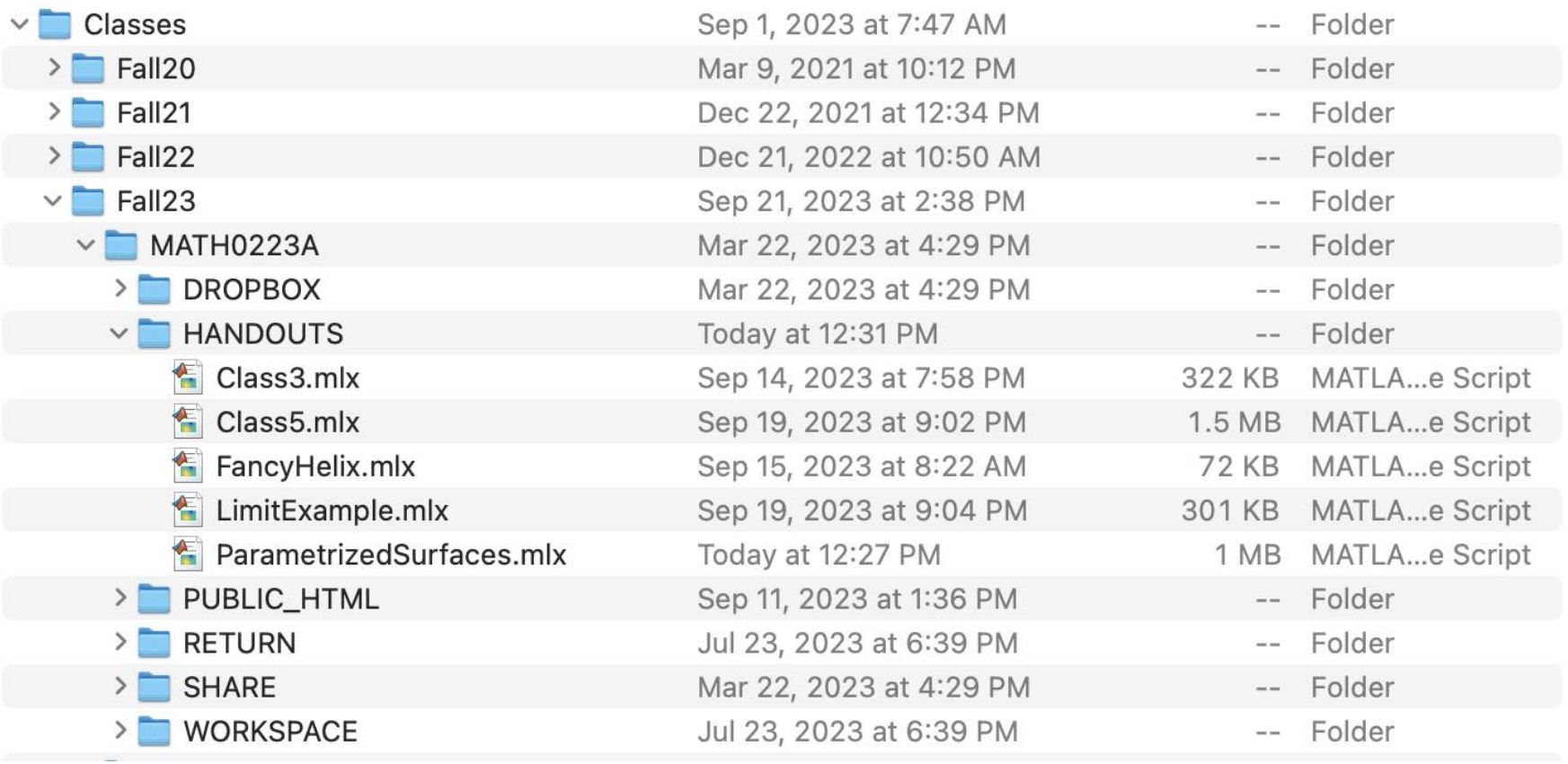
No Books,Computers, Smart Phones, etc.

One Page of Your Own Notes OK

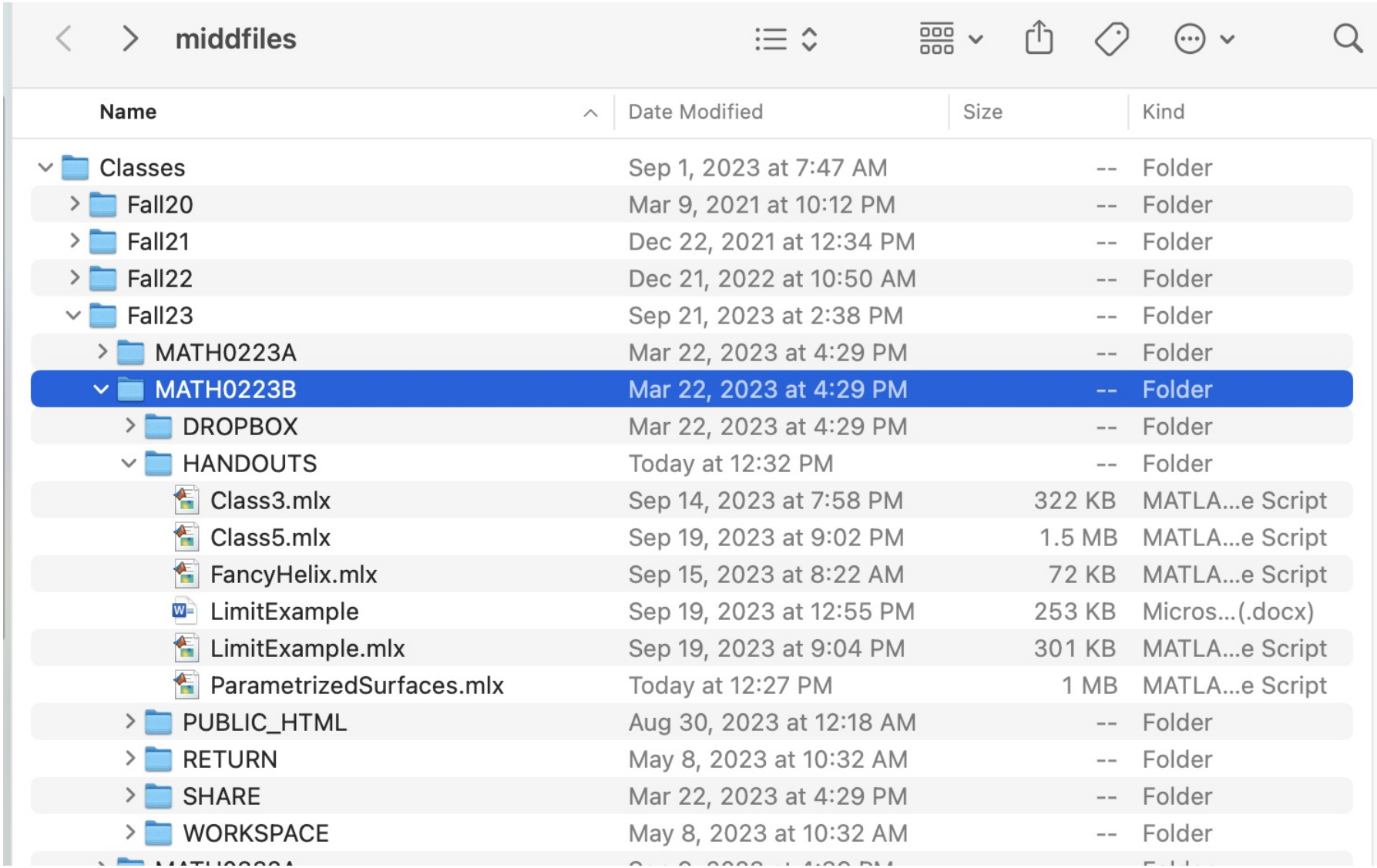
Getting Help

- **My Office Hours**
  Monday, Wednesday, Friday: 10:45 AM to 12 :45 PM
  Or By Appointment
  Warner 202
- **Student Course Tutors**
  Odin Woitek : Tuesdays, 9 to 10 PM
  Megan Paasche: Thursdays, 7 to 9 PM
- **MATLAB Tutors**
  Ai Hattori and Carrie Vanty
  Sunday and Wednesdays, 7 to 9 PM
  Warner 011

MATH 223A: Where to Find the MATLAB Files



MATH 223B: Where to Find the MATLAB Files



Tangent Plane To Graph of $f : \mathcal{R}^n \to \mathcal{R}^1$ at point $(\mathbf{a}, f(\mathbf{a}))$

$$n = 2 : T(\mathbf{x}) = f(\mathbf{a}) + (f_x(\mathbf{a}), f_y(\mathbf{a})) \cdot (\mathbf{x} - \mathbf{a})$$

In general,

$$T(\mathbf{x}) = f(\mathbf{a}) + \nabla f(\mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$$

where $\nabla f(\mathbf{a}) = (f_1)(\mathbf{a}), f_2(\mathbf{a}, ..., f_n(\mathbf{a})$

Tangent Hyperplane
$n = 1$ Ordinary Tangent Line
$n = 2$ Tangent Plane

Example: $f(x,y,z) = \frac{x^2 y}{z}$

Note: $f : \mathcal{R}^3 \to \mathcal{R}^1$ so GRAPH lives in $\mathcal{R}^4$.

Find Equation of Tangent Hyperplane at $\mathbf{a} = (-3, 4, 2)$

$$f_x(x,y,z) = \frac{2xy}{z}$$

$$f_y(x,y,z) = \frac{x^2}{z} \; so \nabla f(x,y,z) = \left(\frac{2xy}{z}, \frac{x^2}{z}, -\frac{x^y}{z^2} \right)$$

$$f_z(x,y,z) = -\frac{x^y}{z^2}$$

at $\mathbf{a} = (-3, 4, 2) : f(\mathbf{a}) = \frac{(-3)^2 \times 4}{2} = 18$

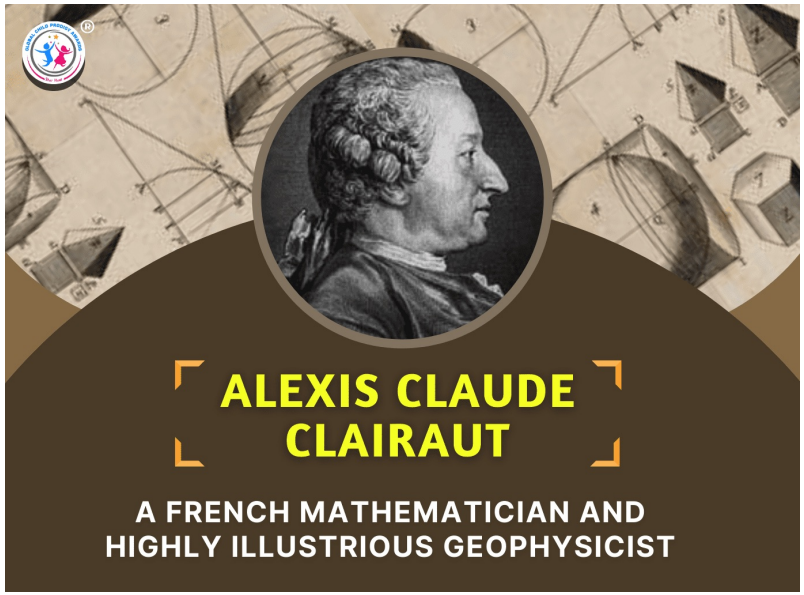
$$\nabla f(\mathbf{a}) = \left(\frac{(2)(-3)(4)}{2}, \frac{(-3)^2}{2}, \frac{-(-3)^2(4)}{2} \right) = \left(-12, \frac{9}{2}, -9 \right)$$

Equation of Tangent Hyperplane is

$$w = 18 + \left(-12, \frac{9}{2}, -9 \right) \cdot (x+3, y-4, z-2)$$

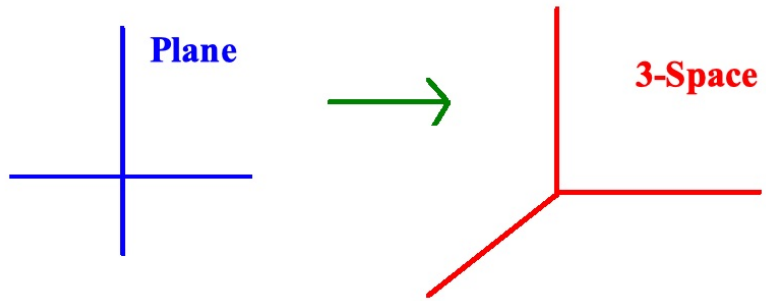
Clairaut's Theorem on Equality of Mixed Partials

If f_{xy} and f_{yx} are continuous at $\mathbf{a}$, then $f_{xy}(\mathbf{a}) = f_{yx}(\mathbf{a})$



May 7, 1713 – May 17, 1765

Parametrized Surfaces



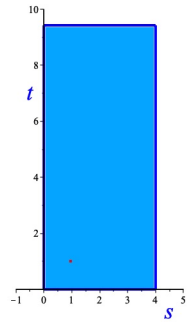
Function from $\mathcal{R}^2 \to \mathcal{R}^3$

Domain:	Patch in Plane
Image:	Surface in Space
Graph:	Lives in $\mathcal{R}^5$

Need for Parametrizations: Graph of $f : \mathcal{R}^1 \to \mathcal{R}^1$ is a curve but not every curve is the graph of such a function

Similarly, graph of $f : \mathcal{R}^2 \to \mathcal{R}^1$ is a surface but not every surface is the graph of such a function.

Example: $\sigma(s,t) = (s\cos t, s\sin t, t), 0 \le s \le 4, 0 \le t \le 3\pi$

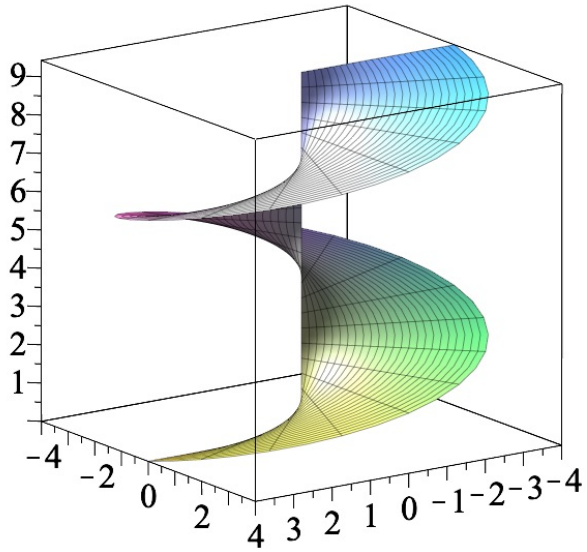


Point: $(1, \pi/4)$ so $\sigma(1,\pi/4) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

$\sigma_s(s,t) = (\cos t, \sin t, 0)$ and $\sigma_t(s,t) = (-s\sin t, s\cos t, 1)$

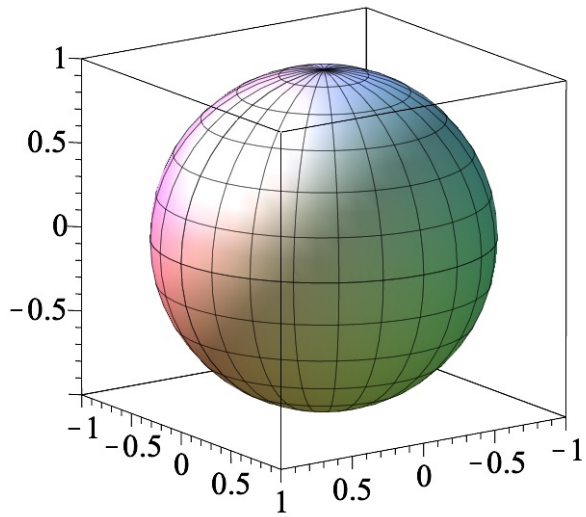
At $\left(1, \frac{\pi}{4}\right)$, representation of the tangent plane is

$$\sigma\left(1,\frac{\pi}{4}\right) + \sigma_s\left(1,\frac{\pi}{4}\right)s + \sigma_t\left(1,\frac{\pi}{4}\right)t$$



Parametrize Unit Sphere

$$\sigma(s,t) = (\cos t \cos s, \sin t \cos s, \sin s), 0 \leq s \leq 2\pi, 0 \leq t \leq 2\pi$$



$$x = \cos t \cos s, y = \sin t \cos s, z = \sin s$$

$$\begin{aligned} x^2 + y^2 + z^2 &= \cos^2 t \cos^2 s + \sin^2 t \cos^2 s + \sin^2 s \\ &= \cos^2 s(\cos^2 t + \sin^2 t) + \sin^2 s \\ &= \cos^2 s + \sin^2 s = 1 \end{aligned}$$

Parametrize Cylinder

$$x = s, y = 4\cos t, z = 4\sin t, 0 \leq s \leq 3, 0 \leq t \leq 2\pi$$