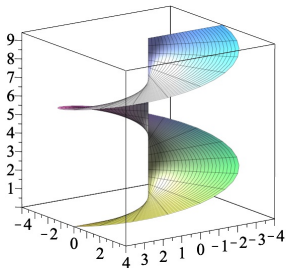


# MATH 223: Multivariable Calculus



Class 8: Wednesday  
September 27, 2023



- ▶ Notes on Assignment 7
- ▶ Assignment 8
- ▶ Unified Treatment Of Tangent Lines and Planes
- ▶ Parametrized Surfaces in *MATLAB* [Handouts Folder]

# Announcements

Exam 1: Next Wednesday, 7 PM -  
No Time Limit

No Books, Computers, Smart Phones,  
etc.

**One Page of Your Own Notes  
OK**

## Getting Help

- ▶ **My Office Hours**

Monday, Wednesday, Friday: 10:45 AM to 12 :45 PM

Or By Appointment

Warner 202

- ▶ **Student Course Tutors**

Odin Woitek : Tuesdays, 9 to 10 PM

Megan Paasche: Thursdays, 7 to 9 PM






- ▶ **MATLAB Tutors**

Ai Hattori and Carrie Vanty

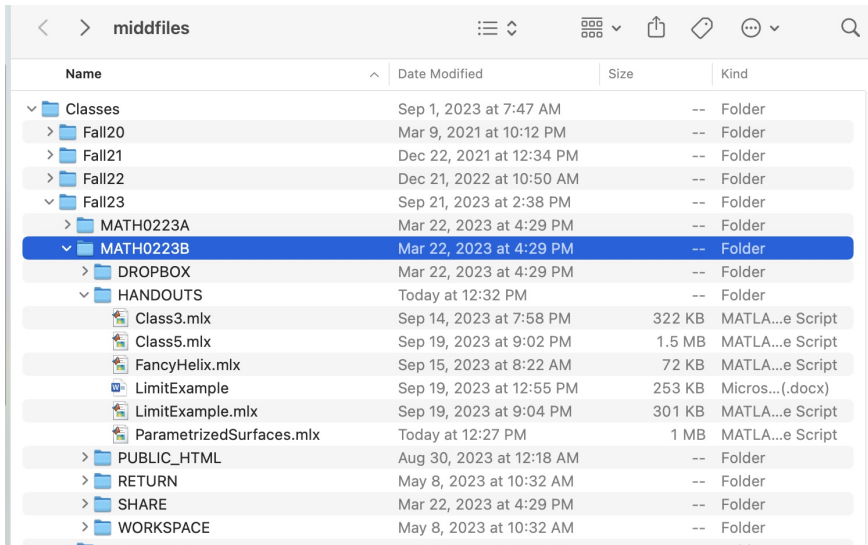
Sunday and Wednesdays, 7 to 9 PM

Warner 011

## MATH 223A: Where to Find the MATLAB Files

▼	Classes	Sep 1, 2023 at 7:47 AM	--	Folder
>	Fall20	Mar 9, 2021 at 10:12 PM	--	Folder
>	Fall21	Dec 22, 2021 at 12:34 PM	--	Folder
>	Fall22	Dec 21, 2022 at 10:50 AM	--	Folder
▼	Fall23	Sep 21, 2023 at 2:38 PM	--	Folder
▼	MATH0223A	Mar 22, 2023 at 4:29 PM	--	Folder
>	DROPBOX	Mar 22, 2023 at 4:29 PM	--	Folder
▼	HANDOUTS	Today at 12:31 PM	--	Folder
	 Class3.mlx	Sep 14, 2023 at 7:58 PM	322 KB	MATLA...e Script
	 Class5.mlx	Sep 19, 2023 at 9:02 PM	1.5 MB	MATLA...e Script
	 FancyHelix.mlx	Sep 15, 2023 at 8:22 AM	72 KB	MATLA...e Script
	 LimitExample.mlx	Sep 19, 2023 at 9:04 PM	301 KB	MATLA...e Script
	 ParametrizedSurfaces.mlx	Today at 12:27 PM	1 MB	MATLA...e Script
>	PUBLIC_HTML	Sep 11, 2023 at 1:36 PM	--	Folder
>	RETURN	Jul 23, 2023 at 6:39 PM	--	Folder
>	SHARE	Mar 22, 2023 at 4:29 PM	--	Folder
>	WORKSPACE	Jul 23, 2023 at 6:39 PM	--	Folder

## MATH 223B: Where to Find the MATLAB Files



The screenshot shows a file explorer window titled "middfiles". The main content is a table listing files and folders. The "MATH0223B" folder is selected and highlighted in blue. Below it, several MATLAB script files (.mlx) and a Microsoft Word document (.docx) are listed.

Name	Date Modified	Size	Kind
Classes	Sep 1, 2023 at 7:47 AM	--	Folder
> Fall20	Mar 9, 2021 at 10:12 PM	--	Folder
> Fall21	Dec 22, 2021 at 12:34 PM	--	Folder
> Fall22	Dec 21, 2022 at 10:50 AM	--	Folder
> Fall23	Sep 21, 2023 at 2:38 PM	--	Folder
> MATH0223A	Mar 22, 2023 at 4:29 PM	--	Folder
> <b>MATH0223B</b>	<b>Mar 22, 2023 at 4:29 PM</b>	--	<b>Folder</b>
> DROPBOX	Mar 22, 2023 at 4:29 PM	--	Folder
> HANDOUTS	Today at 12:32 PM	--	Folder
Class3.mlx	Sep 14, 2023 at 7:58 PM	322 KB	MATLA...e Script
Class5.mlx	Sep 19, 2023 at 9:02 PM	1.5 MB	MATLA...e Script
FancyHelix.mlx	Sep 15, 2023 at 8:22 AM	72 KB	MATLA...e Script
LimitExample	Sep 19, 2023 at 12:55 PM	253 KB	Micros...(docx)
LimitExample.mlx	Sep 19, 2023 at 9:04 PM	301 KB	MATLA...e Script
ParametrizedSurfaces.mlx	Today at 12:27 PM	1 MB	MATLA...e Script
> PUBLIC_HTML	Aug 30, 2023 at 12:18 AM	--	Folder
> RETURN	May 8, 2023 at 10:32 AM	--	Folder
> SHARE	Mar 22, 2023 at 4:29 PM	--	Folder
> WORKSPACE	May 8, 2023 at 10:32 AM	--	Folder

Tangent Plane To Graph of  $f : \mathcal{R}^n \rightarrow \mathcal{R}^1$  at point  $(\mathbf{a}, f(\mathbf{a}))$

$$n = 2 : T(\mathbf{x}) = f(\mathbf{a}) + (f_x(\mathbf{a}), f_y(\mathbf{a})) \cdot (\mathbf{x} - \mathbf{a})$$

In general,

$$T(\mathbf{x}) = f(\mathbf{a}) + \nabla f(\mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$$

where  $\nabla f(\mathbf{a}) = (f_1(\mathbf{a}), f_2(\mathbf{a}), \dots, f_n(\mathbf{a}))$

Tangent Hyperplane

$n = 1$  Ordinary Tangent Line

$n = 2$  Tangent Plane

Example:  $f(x, y, z) = \frac{x^2y}{z}$

Note:  $f : \mathcal{R}^3 \rightarrow \mathcal{R}^1$  so GRAPH lives in  $\mathcal{R}^4$ .

Find Equation of Tangent Hyperplane at  $\mathbf{a} = (-3, 4, 2)$

$$f_x(x, y, z) = \frac{2xy}{z}$$

$$f_y(x, y, z) = \frac{x^2}{z} \text{ so } \nabla f(x, y, z) = \left( \frac{2xy}{z}, \frac{x^2}{z}, -\frac{xy}{z^2} \right)$$

$$f_z(x, y, z) = -\frac{xy}{z^2}$$

at  $\mathbf{a} = (-3, 4, 2) : f(\mathbf{a}) = \frac{(-3)^2 \times 4}{2} = 18$

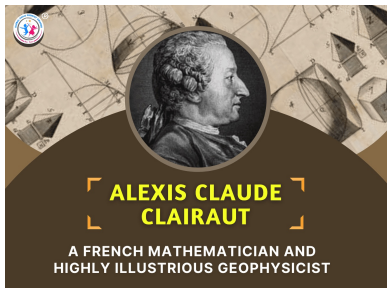
$$\nabla f(\mathbf{a}) = \left( \frac{(2)(-3)(4)}{2}, \frac{(-3)^2}{2}, \frac{-(-3)^2(4)}{2} \right) = \left( -12, \frac{9}{2}, -9 \right)$$

Equation of Tangent Hyperplane is

$$w = 18 + \left( -12, \frac{9}{2}, -9 \right) \cdot (x + 3, y - 4, z - 2)$$

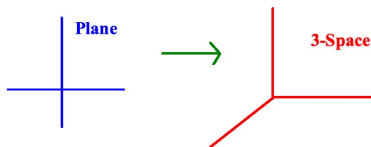


Clairaut's Theorem on Equality of Mixed Partial  
If  $f_{xy}$  and  $f_{yx}$  are continuous at  $\mathbf{a}$ , then  $f_{xy}(\mathbf{a}) = f_{yx}(\mathbf{a})$



May 7, 1713 – May 17, 1765

## Parametrized Surfaces



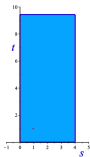
Function from  $\mathcal{R}^2 \rightarrow \mathcal{R}^3$

Domain: Patch in Plane  
Image: Surface in Space  
Graph: Lives in  $\mathcal{R}^5$

Need for Parametrizations: Graph of  $f : \mathcal{R}^1 \rightarrow \mathcal{R}^1$  is a curve but not every curve is the graph of such a function

Similarly, graph of  $f : \mathcal{R}^2 \rightarrow \mathcal{R}^1$  is a surface but not every surface is the graph of such a function.

Example:  $\sigma(s, t) = (s \cos t, s \sin t, t), 0 \leq s \leq 4, 0 \leq t \leq 3\pi$

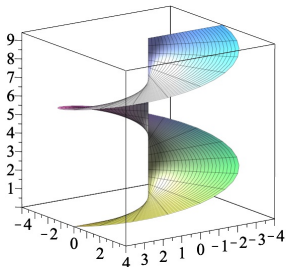


Point:  $(1, \pi/4)$  so  $\sigma(1, \pi/4) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$

$\sigma_s(s, t) = (\cos t, \sin t, 0)$  and  $\sigma_t(s, t) = (-s \sin t, s \cos t, 1)$

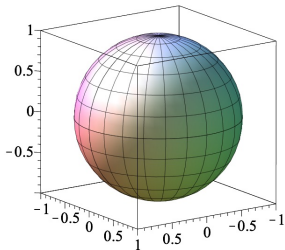
At  $(1, \frac{\pi}{4})$ , representation of the tangent plane is

$$\sigma\left(1, \frac{\pi}{4}\right) + \sigma_s\left(1, \frac{\pi}{4}\right) s + \sigma_t\left(1, \frac{\pi}{4}\right) t$$



## Parametrize Unit Sphere

$$\sigma(s, t) = (\cos t \cos s, \sin t \cos s, \sin s), 0 \leq s \leq 2\pi, 0 \leq t \leq 2\pi$$



$$x = \cos t \cos s, y = \sin t \cos s, z = \sin s$$

$$\begin{aligned}x^2 + y^2 + z^2 &= \cos^2 t \cos^2 s + \sin^2 t \cos^2 s + \sin^2 s \\&= \cos^2 s (\cos^2 t + \sin^2 t) + \sin^2 s \\&= \cos^2 s + \sin^2 s = 1\end{aligned}$$

## Parametrize Cylinder

$$x = s, y = 4 \cos t, z = 4 \sin t, 0 \leq s \leq 3, 0 \leq t \leq 2\pi$$

