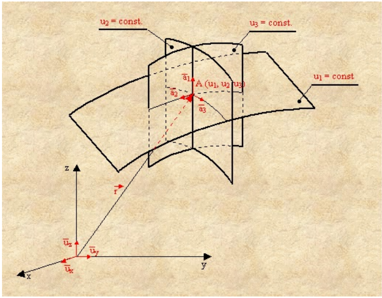
MATH 223: Multivariable Calculus



Class 21: October 30, 2023


Happy
Halloween

Christian Stratton, Montana State University
Tuesday, October 31 · 12:30–1:20 PM · Warner 101

“An overview of Bayesian statistical methodology for ecological and environmental applications”

With a changing global climate, there is heightened interest in assessing how plant and animal communities respond to various stressors, including climate change and wildlife disease. Simultaneously, modern technology has enabled unprecedented volumes of monitoring data, requiring the development of new statistical methodologies to accommodate these data streams and address pressing conservation needs. We will provide an overview of two ecological problems and describe the development of statistical methodology to address these problems. The first topic concerns an assessment of the impact of invasive species on native vegetation communities in Craters of the Moon National Monument and Preserve in Idaho, USA. The second topic concerns assessing the impact of a wildlife disease known as White-nose syndrome on susceptible bat populations in Montana, USA. We will focus on the motivation for statistical development, providing necessary prerequisite information on the ecological processes of interest in an interactive format.

Maddie Rainey, Colorado State University
Thursday, November 2 · 12:30–1:20 PM · Warner 101

“Modeling Spatial Data with the Flexibility of Spatial Splines”

Spatially-varying data are at the core of many disciplines such as epidemiology, criminology, geography, and astronomy. I will introduce two real-world examples of spatial data and basic methods for visualizing and manipulating spatial data in R. I will then introduce spatial splines and show how individual splines are combined in an analysis to model the spatial variability. A visual demonstration will show how splines can efficiently model 1-dimensional and 2-dimensional surfaces more flexibly than strictly parametric approaches. I conclude with an overview of an epidemiological application of spatial splines in R for a regression analysis.

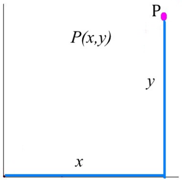
Notes on Assignment 19
Assignments 20 and 21
Curvilinear Coordinates

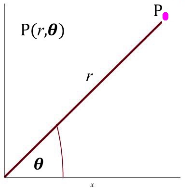
Announcements

Exam 2: Wednesday
7 PM –?
Warner 101

Review Basic Theorems About Integration from Calculus I

Today:
Curvilinear Coordinates
Coordinate Systems in Plane and Space
Plane





Cartesian **Polar**

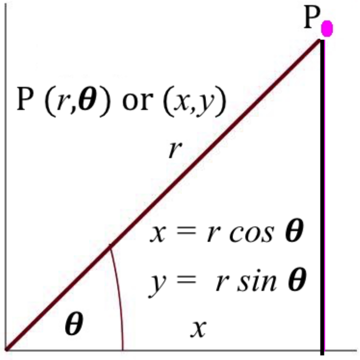
Newton(1671) [Not published until after death]
Jacob Bernoulli (1691)

In Polar Coordinates, Circles and Lines Through Origin Have Simple Equations:

Circle: $r = 4$

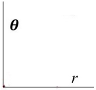
Line: $\theta = \pi/6$

Relationship Between Polar and Cartesian

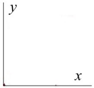


Linear Algebra Perspective

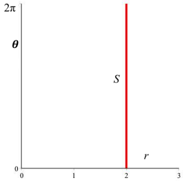
$$P\begin{pmatrix} r \\ \theta \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \begin{matrix} \text{for } 0 < r < \infty \\ 0 \le \theta \le 2\pi \end{matrix}$$

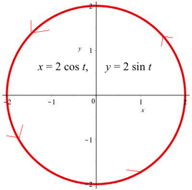


$$P : U^2 \to \mathbb{R}^2$$



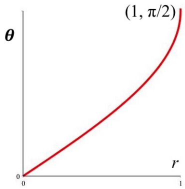
<u>Example</u>: P sends this line segment S to circle of radius 2 with center at origin.





$$[r, \theta] = [2, t], 0 \le t \le 2\pi$$

Example: $\theta = t, r = \sin t = \sin\theta$



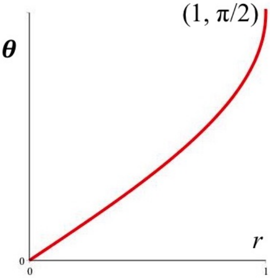
	$r = \sin t$	$\theta = t$
0	0	0
$\pi/6$	$1/2$	$\pi\ /6$
$\pi/4$	$\sqrt{2}/2$	$\pi/4$
$\pi/2$	1	$\pi/2$

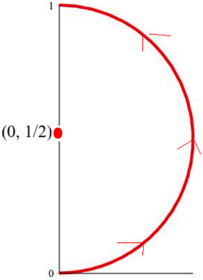
Then $x = r\cos\theta = \sin\theta\cos\theta$ and $y = r\sin\theta = \sin\theta\sin\theta$
So $x^2 = \sin^2\theta\cos^2\theta, y^2 = \sin^2\theta\sin^2\theta$
and then $x^2 + y^2 = \sin^2\theta(\cos^2\theta + \sin^2\theta) = \sin^2\theta \times 1 = \sin^2\theta = y$
Thus $x^2 + y^2 - y = 0$.
Complete the square in y:
$x^2 + y^2 - y + \frac{1}{4} = \frac{1}{4} \implies x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$
which is the equation of a circle with center at (0,1/2) and radius 1/2.

$$x^2 + y^2 - y + \frac{1}{4} = \frac{1}{4} \implies x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$$

which is the equation of a circle
with center at (0,1/2) and radius 1/2.

The image is the right half of the circle:





Think of P as a function from $\mathbb{R}^2$ to $\mathbb{R}^2$. Then

$$P' = \begin{pmatrix} \frac{\partial}{\partial r}(r\cos\theta) & \frac{\partial}{\partial \theta}(r\cos\theta) \\ \frac{\partial}{\partial r}(r\sin\theta) & \frac{\partial}{\partial \theta}(r\sin\theta) \end{pmatrix}$$

$$P' = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} \implies P'(\pi/6) = \begin{pmatrix} \sqrt{3}/2 & -1/4 \\ 1/2 & \sqrt{3}/4 \end{pmatrix}$$

Previous Example

$$g(t) = [\sin t, t]$$

$$\text{so } g'(t) = [\cos t, 1]$$

$$\text{At } t = \pi/6, g'(\pi/6) = \left[\sqrt{3}/2, 1\right]$$

$$g : \mathbb{R}^1 \to \mathbb{U}^2 \text{ and } P : \mathbb{U}^2 \to \mathbb{R}^2$$

$$(P \circ g) : \mathbb{R}^1 \to \mathbb{R}^2$$

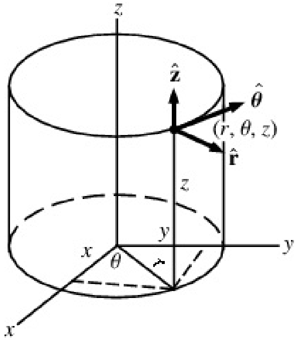
$$(P \circ g)' = P'(g) \cdot g'$$

Evaluate at $\pi/6$:

$$\begin{pmatrix} \sqrt{3}/2 & -1/4 \\ 1/2 & \sqrt{3}/4 \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix}$$

Coordinate Systems in 3-Space

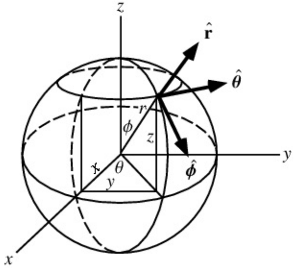
Cylindrical Coordinates: (r, θ, z).



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

Coordinate Systems in 3-Space

Spherical Coordinates: $(\rho, \theta, \phi) = (r, \theta, \phi)$

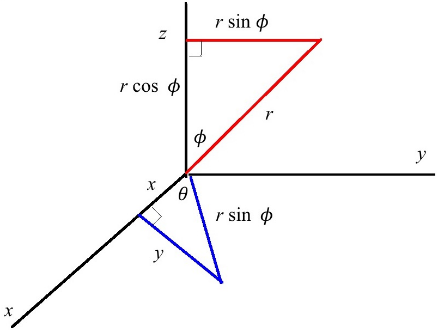


$r =$ distance between origin and point
$\theta =$ project down to xy-plane
$\phi =$ rotation down from vertical axis

$r =$ distance between origin and point	$x = r \sin \phi \cos \theta$
$\theta =$ project down to xy-plane.	$y = r \sin \phi \sin \theta$
$\phi =$ rotation down from vertical axis	$z = r \cos \phi$

Some authors use ρ instead of r.

Converting from Spherical To Cartesian

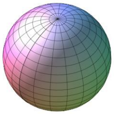


$$\cos\phi = \frac{z}{r} \implies z = r\cos\phi$$

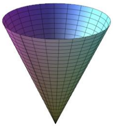
$$\cos\theta = \frac{x}{r\sin\phi} \implies x = r\sin\phi\cos\theta$$

$$\sin\theta = \frac{y}{r\sin\phi} \implies y = r\sin\phi\sin\theta$$

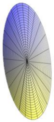
$$S : \begin{pmatrix} r \\ \phi \\ \theta \end{pmatrix} \to \begin{pmatrix} r \sin\phi \cos\theta \\ r \sin\phi \sin\theta \\ r \cos\phi \end{pmatrix}$$



$r =$ Constant
Sphere



$\phi =$ Constant
Cone



$\theta =$ Constant
Plane

Jacobian Matrices

$$\begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \sin\phi\cos\theta & r\cos\phi\cos\theta & -r\sin\phi\sin\theta \\ \sin\phi\sin\theta & r\cos\phi\sin\theta & r\sin\phi\cos\theta \\ \cos\phi & -r\sin\phi & 0 \end{pmatrix}$$