MATH 223

Some Hints and Answers for Assignment 27

Exercises 9ac, 10df, 13 and 16 of Chapter 7.

9ac: Compute the indicated line integrals $\int_\gamma \mathbf{F}$ for the given vector fields and parametrizations $\mathbf{g}$ for γ

(a) $\mathbf{F} = (x, 2y, 3z), \mathbf{g}(t) = (t, t, t), 0 \le t \le 2$: $\int_0^2 (t, 2t, 3t) \cdot (1, 1, 1)\, dt = \int_0^2 6t\, dt = 12$.

(b)$\int_0^\pi -\sin t + \cos t\, dt = [\cos t + \sin t]_0^\pi = -2$.

10df: Sketch the curves γ_1 and γ_2 given respectively by the parametrizations $\mathbf{g_1}(t) = (\cos t, \sin t), 0 \le t \le \pi/2$ and $\mathbf{g_2}(s) = (1 - s, s), 0 \le s \le 1$.
Hint: One curve is a quarter of a circle; the other is a line segment.
Then determine the line integral of the vector field $\mathbf{F}$ along both γ_1 and γ_2 for each of the following vector fields:

(d) $\mathbf{F}(x, y) = (x, y)$: $\int_{\gamma_1} \mathbf{F} = 0$ and $\int_{\gamma_2} \mathbf{F} = 0$.

(f) $\mathbf{F}(x, y) = (xy^2, x^2y)$: Both line integrals are also 0.

11: Calculate the line integral of the gradient of the Cobb-Douglas function $f(K, L) = 2K^{2/3}L^{1/3}$ of Example 2 along

(a) the straight line between (1,1) and (27, 64)
Use the Fundamental Theorem, value is 70.

(b) the level curve where $f(K, L) = 1000$. Answer: 0

13: If $\mathbf{F}$ is the vector field $F(S, I, R) = (-\beta SI, \beta SI - rI, rI)$, described in the SIR Epidemic Model, find the line integral of $\mathbf{F}$ along the curve γ with parametrization $\mathbf{g}(t) = (t^2, t, t^2), 100 \le t \le 200$.
Note: You may wind up with the integral $\int_{100}^{200} -2\beta t^4 + \beta t^3 - rt + 2rt^2\, dt$ which equals $-123625000000\beta + \frac{13955000}{3} r$.

16: Let $\mathbf{F}$ be the vector field defined by $\mathbf{F}(x, y) = (-y^2, 2x^2)$.

(a) Sketch the vector field.
See the Assignment 27 sheet.

(b) Find two different curves g_1 and g_2 connecting the points (0,0) and (2,4) so that the line integrals of $\mathbf{F}$ along the curves are different.
Hint: Consider straight lines and parabolas.

(c) Find a closed curve γ in the plane where the starting point is the same as the ending point but $\int_\gamma \mathbf{F}$ is not zero.
Solution: Let γ be the path that starts at (0,0), runs along g_1 to (2,4) and then runs back along g_2 to (0,0).

(d) Explain why parts (b) and (c) each allow you to conclude that $\mathbf{F}$ is not a gradient field.
Hint: What must be true if it is a gradient field?