MATH 223
Some Hints and Answers for Assignment 26
Exercises 2, 4ad, 5, 7 and 8 of Chapter 7.

2: Work = 4000 newton-meters.

4: Find the work done by the planar vector field $\mathbf{F}(x, y) = (3x + 2y, 2x + 3y)$ along each of following curves:

(a) : $y = x^3$ from (0,0) to (1,1)
Let $g(t) = (t, t^3), 0 \le t \le 1$ parametrize the curve. Show work done = $\int_0^1 3t + 8t^3 + 9t^5\, dt = 5$

(d): $\mathbf{g}(t) = (t^2, t^3), 1 \le t \le 4$
Let $g'(t) = (2t, 3t^2)$ and $F(g(t)) = F(t, t^3) = \left(3t^2 + 2t^3, 2t^2 + 3t^3\right)$, Work = $\int_1^4 6t^3 + 10t^4 + 9t^5\, dt =$ 8571

5: Find the work done by the vector field $\mathbf{F}(x, y, z) = (3x + 2y + z, 2x + 1y + 3z, x + 2y + 3z)$ along the curves

(a) $g(t) = (t, t^2, t^3), 0 \le t \le 1$
Note $\mathbf{F}(g(t)) = (3t+2t^2+t^3, 2t+t^2+3t^3, t+2t^2+3t^3)$. The work done is $\int_0^1 3t+6t^2+6t^3+12t^4+9t^5 dt = \frac{89}{10}$.

(b) $\mathbf{g}(t) = (\cos t, \sin t, t), 0 \le t \le \pi/2$

Show $F(g(t)) \cdot g'(t) = (-3\sin t \cos t - 2\sin t \sin t - t\sin t) + (2\cos t \cos t + \sin t \cos t + 3t\cos t) + (\cos t + 2\sin t + 3t)$. The work done is -2 + $\frac{3}{8}\pi^2 + \frac{3}{2}\pi$.

7: Find a potential function for $\mathbf{F}(x, y) = (2xe^y - \sin x \sin y, x^2e^y + \cos x \cos y)$.
IIntegrate the first component $2xe^y - \sin x \sin y$ with respect to x first.

8: Discuss what happens when you try to find a potential function for $\mathbf{F} = (2xy + y^2, x^2 + 3xy)$
Start with integrating $2xy + y^2$ with respect to x. The result is $x^2y + y^2x + G(y)$ for some function G of y. Then the derivative of this expression with respect to y would be $x^2 + 2yx + G'(y)$.