MATH 223

Hints and Answers for Assignment 15

Exercises 7, 8ab, 10, 11, and 12 in Chapter 5.

7: (b) Since the sine function is bounded by 1 and -1 for all values of x , we have

$$-2x^2 \leq 2x^2 \sin\frac{1}{x} \leq 2x^2.$$

Add x to each term in the inequality

c) Use the definition of the derivative to show $f'(0) = 1$ and that if $x \neq 0$, then. $f'(x) = 1 + 4x \sin\frac{1}{x} - 2\cos\frac{1}{x}$.

Consider the value of $f'(x)$ at $x_1 = \frac{1}{2k\pi}$ and $x_2 = \frac{1}{(2k+1)\pi}$ where k is a nonzero integer. Because 0 is not included in the interval $[x_1, x_2]$, $f'(x)$ is always continuous on this interval. The Intermediate Value Theorem promises that there exists a point c included in $[x_1, x_2]$ such that $f'(c) = 0$. For any arbitrary neighborhood of 0, $\mathcal{M}$, there are infinitely many values of k for which x_1, x_2, and c are all included in $\mathcal{M}$; therefore, any $\mathcal{M}$ includes infinitely many x such that $f'(x) = 0$.

d) Wherever x is non zero we, $f(x)$ is the composition of differentiable functions and is therefore differentiable. Show that f' achieves the value 1 at infinitely many places in an arbitrary neighborhood of 0. The function f' then achieves the values 0 and 1 in any arbitrary neighborhood of 0 and cannot be continuous at $x = 0$. If f' is not continuous at 0 it is also not differentiable there.

11: If a vector field $\mathbf{F}(x, y) = (g(x, y), h(x, y))$ does not have the property $g_y = h_x$, then it is not a Gradient Field.

12:. For $\mathbf{F}(x, y, z)$ to be a gradient field, each second order mixed partial derivative must be equal regardless of order of differentiation. Not only does G_y need to be equal to H_x, but we must have $G_z = K_x$ and $H_z = K_y$.