MATH 223 Multivariable Calculus

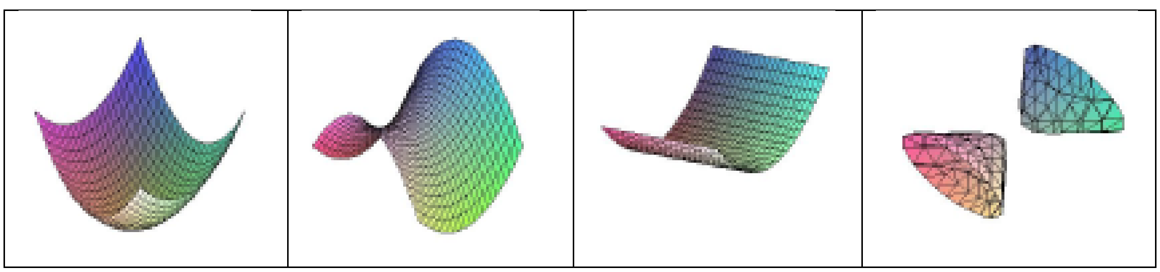
Sample Examination 1

1. An object moves in space in such a way that its position *f(t)* at each time *t* is given by the vector-valued function $\boldsymbol{f}(t) = (\cos t, \ln(1+t^2), e^{-2t})$
Compute each of the following

(a) $\boldsymbol{f}'(t)$
(b) $\boldsymbol{f}''(t)$
(c) Position at $t = 0$
(d) Velocity at $t = 0$
(e) Speed at $t = 0$
(f) Parametric equation for the tangent line to the curve at $t = 0$, and
(g) The dimension of the image of *f*.

2. An object moves in space in such a way that its position is given by some twice differentiable vector-valued function **p**(t) in such a way that its *speed* has a constant value of 4. Show that velocity and acceleration vectors are always orthogonal.

3. Let g: $\mathbb{R}^2 \to \mathbb{R}^1$ be given by $g(x,y) = 2x^2 - 3y^2$. Sketch the level curves in $\mathbb{R}^2$ for
(a) $g(x,y) = 18$
(b) $g(x,y) = 0$
(c) $g(x,y) = -1$
Circle the picture below which best represents the graph of g:



4. In 1938, two future winners of the Nobel Prize in Economics, Ragnar Frisch and Trygve Haavelmo, published a paper "Ettersporselen etter melk i Norge" ("The Demand for Milk in Norway") They found that the milk production *z* is related to *p*, the relative price of milk, and *r*, the income per family through the equation

$$z = f(r,p) = k\frac{r^a}{p^b}$$ where *k, a* and *b* are positive constants.

Suppose $b = 3/2$ and $a = 2$. (Frisch and Haavelmo actually found $a = 2.08$).
(a) Use the ***definition*** of partial derivatives to find $\frac{\partial z}{\partial r} = f_r$ at the point where $r = 3$, $p = 4$
(b) Find gradient $\nabla f(x,y)$ if $f(x,y) = k\,\frac{x^2}{y^{3/2}}$ for some constant *k*.
(c) Find an equation for the tangent plane at (3, 4, *f*(3,4)) to the surface $z = f(x,y)$

5. Let f be the real-valued function of two variables defined by

$$f(x,y) = \begin{cases} \dfrac{xy}{ax^2+by^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

where a is number of the month and b is the day on the month on which you were born. For example, if your birthday is October 4, you would use $a = 10$, $b = 4$ for the rest of this problem.

(a) Find the limit of f as (x,y) approaches (0,0) along the line $y = x$.
(b) Find the limit of f as (x,y) approaches (0,0) along the line $y = -x$.
(c) Prove that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.
(d) Find the maximum value **M** of $f(x,y)$.
(e) What subset of the real numbers is the image of this function?

6. (a) Find the functions f_{xy}, f_{zy}, and f_{xyz}, if $f(x,y,z) = \frac{x^2y}{z}$.
(b). Show that the function given by $f(s,t) = (s\cos t, s\sin t, s)$ for $0 \le s \le 4$, $0 \le t \le 2\pi$ twists a rectangle in the (s,t)-plane into a piece of the surface in 3-dimensional space satisfying the equation $x^2 + y^2 = z^2$. Sketch and describe what that surface looks like. Provide a clear explanation of your reasoning.