MATH 223 Fall 2022
Assignment 6
Due: Monday, September 25

Reading

Read carefully Sections 3.4 "Parametrized Surfaces" in our text *Multivariable Calculus: A Linear Algebra Based Approach.*

Writing

Write out careful and complete solutions of Exercise 12 in Chapter 3 and the exercises below. (If you have the earlier version of the text, this is Exercise 10)

1. In class, we examined the function given by $f(x,y) = \frac{xy}{x^2+2y^2}$.

(a) Show that 3/19 is a possible value for this function by exhibiting a specific point (*a*,*b*) such that $f(a,b) = \frac{3}{19}$.

(b) Show that there is no point (*a*,*b*) such that $f(a,b) = 1$.

(c) What is the largest possible value **M** of this function?

[*Hint* for this problem: every point in the plane not on the vertical axis lies on a line with equation $y = mx$ for some constant m.]

2. Let g be the function defined by $g(x,y) = \frac{xy}{2x^2+3y^2}$.

(a) What is the domain of this function?

(b) Show that $\lim_{(x,y)\to(0,0)} g(x,y)$ does not exist.

(c) For which points (*x*,*y*) in the plane is $g(x,y) > 0$? For which points if $g(x,y) < 0$?

(d) What is the image of this function? You should carefully describe the set of real numbers which are possible values for this function.

3. Let f be the function defined by $g(x,y) = \frac{x^2y}{x^4+y^2}$.

(a) Show that the limit of f as (*x*,*y*) approaches the origin along any line is 0.

(b) Show that the limit of f as (*x*,*y*) approaches the origin along the curve $y = x^2$ is ½.

(c) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Justify your answer.