

MATH 223 Fall 2023
Assignment 16
Due: Monday, October 23

Reading

Read carefully Section 5.5 “Implicit Differentiation” in our text *Multivariable Calculus: A Linear Algebra Based Approach*.

Writing

Write out careful and complete solutions of Exercises 1, 2, and 3 below.

1. Let $f(x, y) = 2x^2 + 3y^2$ be a real-valued function defined on the plane.
 - (a) Let \mathbf{P} be the point $(5,4)$ and C the level curve of f containing \mathbf{P} . Identify the nature of C : Is it a Circle? Parabola? Pair of Lines? Sketch a picture of C .
 - (b) Use classic implicit differentiation or some other method to find an equation for the line L tangent to C at \mathbf{P} .
 - (c) Determine the gradient vector \mathbf{v} of f at \mathbf{P} .
 - (d) Show that this gradient vector is orthogonal to any vector lying along L .
2. Let $g(x, y) = 2x^2 - 3y^2$ be a real-valued function defined on the plane.
 - (a) Let \mathbf{P} be the point $(5,4)$ and C the level curve of g containing \mathbf{P} . Identify the nature of C : Is it a Circle? Parabola? Pair of Lines? Sketch a picture of C .
 - (b) Use classic implicit differentiation or some other method to find an equation for the line L tangent to C at \mathbf{P} .
 - (c) Determine the gradient vector \mathbf{v} of g at \mathbf{P} .
 - (d) Show that this gradient vector is orthogonal to any vector lying along L .
3. (Williamson and Trotter) A spaceship traveling in the plane along a path such that at time $t \geq 0$, the ship is at position $g(t) = (3t^2, t^3)$. The intensity of gamma radiation at the point (x,y) in the plane is $I(x, y) = x^2 - y^2$, wherever $I(x, y) \geq 0$. Describe fully, using a labeled graph where appropriate, the following:
 - (a) The level curve of I the ship is on at $t = 1$.
 - (b) The path of the ship for $t \geq 0$.
 - (c) The gradient vector of I at the ship's position when $t = 1$.
 - (d) The ship's velocity vector at $t = 1$.
 - (e) The time – if there is one – when the ship stops increasing its radiation risk and begins its race to safety. Does its course become more dangerous later on?