

MATH 223 Fall 2023  
Assignment 11  
Due: Monday, October 9

**Reading**

Read carefully Sections 4.3 “Directional Derivatives” in our text *Multivariable Calculus: A Linear Algebra Based Approach*.

**Writing**

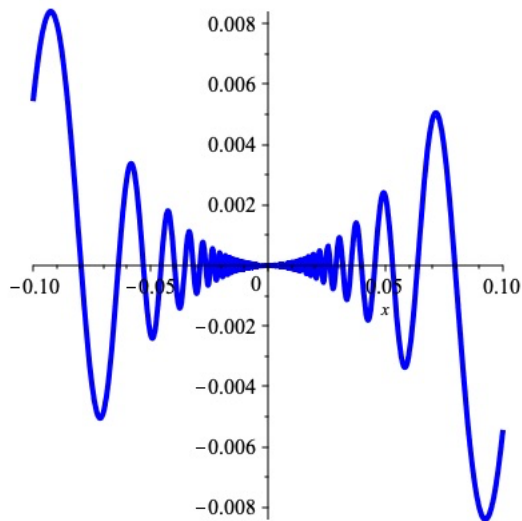
Write out careful and complete solutions of Exercises 17 and 18 in Chapter 4 as well as Problems A, B, and C below.

*Problem A:* For each of these functions  $f$  find gradient  $\nabla f(\mathbf{x})$  of  $f$  at a general point in the domain of  $f$ :

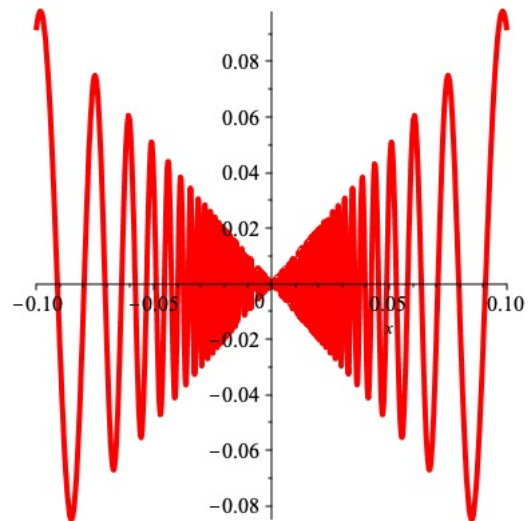
- (1)  $f(x, y) = 2x^3 - 3y^2$
- (2)  $f(x, y, z) = (5x - 7y)z$
- (3)  $f(x_1, x_2, x_3) = \frac{x_1 x_3}{x_2}$

*Problem B:* Write an equation in terms of the coordinate variables  $(x, y, z)$  for the tangent hyperplane for  $f(x, y, z) = 2x^2 - y^2 + 3z^2$  when  $x = y = z = 1$ .

*Problem C:* Let  $f$  be the real-valued function  $f: \mathbb{R}^p \rightarrow \mathbb{R}$  defined by  $f(\mathbf{x}) = |\mathbf{x}|^2 = \mathbf{x} \cdot \mathbf{x}$ . If  $p = 2$ , prove that  $\nabla f(\mathbf{x}) = 2\mathbf{x}$  for all  $\mathbf{x}$  in  $\mathbb{R}^p$ . Is this result true for other values of  $p$ ?



Graph of  
 $f(x) = x^2 \sin\left(\frac{1}{x}\right)$



Graph of  
 $f(x) = 2x \sin\left(\frac{1}{x}\right)$