Math 335 - Differential Geometry

Fall 2023

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Office Hours: M 9:00-10:00am, Tu 1:00-2:30pm, Th 11:15am-12:45pm, and by appointment

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Course Description

In this course, we will introduce ourselves to differential geometry by studying curves and surfaces in $\mathbb{R}^3$. Intuitively we know that a helix is different from a straight line and that a sphere is different from a plane or a saddle. We will make these distinctions rigorous by defining *curvature*. Once we make formal the differences between these surfaces, we can study *geodesics*, which are straight lines from the point of view of the surface. At the end of the semester, we will learn about the Gauss-Bonnet theorem, a beautiful theorem that relates the sum of the angles of a geodesic triangle on a surface to the curvature of the surface.

Course Website

The website for this course can be found at:

http://f23.middlebury.edu/MATH0335A/

This is the central location for our course. Here you will find all course information, handouts, and homework assignments.

Text

We will cover most of Chapters 1-8 in *A First Course in Differential Geometry* by Woodward and Bolton.

Course Objectives

The goals of this course are to help you develop:

- an understanding of the concepts, notation, and theory of differential geometry,
- the ability to use your understanding to solve straightforward and complex problems,
- proof-writing and oral communication skills,
- skills in collaboration, and
- a deepening understanding of the activity of learning.

Course Structure

Our course structure is based on the unit of a week. For each week (MWF) there will be a reading assignment and a problem set. Over the course of the week, I will also post a series of videos that explain the material in the reading. The problem set for each week's material will be due on Monday of the following week.

We will use our class time to work together in groups on homework problems, and to make connections as a class between the homework and the course material. In order to keep us all on track, at the time that I post the homework each week, I will select one or two problems for you to work on before each class meeting time. Please make sure to try these problems ahead of time so that class time will be useful and meaningful for you and your classmates.

Our class time is also meant to give you a regular opportunity to practice speaking about our course material. Our midterm exam and final exam will each have an oral component to it. Class time is a time for you to prepare for those.

Attendance

I expect you to attend all classes. Your attentive presence in class obviously benefits you, but it also makes a significant contribution to class as a whole.

Homework

Homework assignments will be assigned weekly, posted on the course website. You will use LaTeX to typeset and create a .pdf file for each assignment. The homework for each week (MWF) will be collected in class on the following Monday.

Differential geometry has a beautiful visual component to it, and it is at the same time notationally heavy. In order to learn and develop intuition for the notions of differential geometry, *it is essential that you work carefully through each homework assignment.* Each assignment will involve a number of proofs so that you can also work on your mathematical writing skills. When writing, aim for clean, logically-ordered, and clearly-written arguments that would be understandable to your peers.

Along with working problems, *watching the videos and reading the course notes are important components of your homework.* Be an active participant: watch and read with a paper and pencil. Work out details as you read and frequently ask yourself such questions as "Where was the hypothesis used in this proof?" and "Do I understand each individual line in this proof as well as the general flow of the overall argument?" Learning will sometimes feel slow, and often frustrating. That is normal. The payoff is that it can be deeply satisfying to understand something if you've had to work at it for a while first. Plus, the video, notes, and book will provide you with many examples of proofs after which you can model your own.

Our time in class is designed to be a support to you with your problem sets. I will select some problems, some routine and some more challenging, for us to focus on during our class time. You should attempt these problems as completely as you can before class, but you can use our class time to work together with classmates and to get help from me.

Homework exercises will be marked on a 0-5 scale. A 5 is given for a clearly-written and consistent answer, scores of 1-4 are given for answers that have some logical flaws or that need more explanation, and a score of 0 is given when no attempt is made to answer the problem. In order to help you focus on and improve in your writing, one point of each graded problem will be devoted simply to the clarity of your expression. Notice that homework is a relatively large percentage of your final grade so this should be where a significant amount of your energy for the class goes.

Note that **no late homework will be accepted**. Late homework will receive a score of zero. You may, however, hand in homework early if a conflict arises. Given that life often happens in ways that are out of our control, I will drop your lowest homework score when calculating your final grade. If you find yourself in a situation that warrants some more leeway, please be in touch so that we can talk about it.

Office Hours

I am here to help. If you have problems with an assignment or anything else that you'd like to discuss about the class, please come see me during my office hours. I will be happy to see you there.

Assessment

There will be a midterm exam during the week of:

Monday, October 23

The final exam will be given during the final exam period. Our midterm and final exams will have both written and oral components.

As with homework, **no late exams will be given or accepted**.

Honor Code

Since homework is meant to be an extension of the type of work that we do in class, I encourage you to work together on your homework assignments. Your final write-up of each assignment, however, must be your own. The best way to achieve this requirement is to **talk** as a group about how to solve a particular problem, but then work on your own as you **write** your formal solution. If you work with other people on your assignments outside of class time, please list the people you worked with at the top of your assignment.

You learn math by doing math, and sometimes the most growth happens during the moment of discomfort when you feel like you don't know what you are doing. With this in mind, even though you may at times feel uncomfortable not knowing how to proceed with a homework problem, the only resources that are permitted when doing your homework assignments are our class videos and notes, your textbook (Woodward and Bolton), each other, and me. The use of any other resources, including AI tools (e.g. ChatGPT), is considered a breach of the Honor Code. For the exams, I will clearly specify the resources you are allowed to use.

The reason for this requirement is that it will allow you to know what you know, and to receive feedback on your assignments that is appropriate to your particular situation. Working in this way also speaks to the important and much broader question of honesty in general. We are not looking for perfection; we are looking for learning.

Students with Disabilities

Students who have Letters of Accommodation in this class are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without Letters of Accommodation, assistance is available to eligible students through the Disability Resource Center (DRC). Please contact ADA Coordinators Jodi Litchfield, Peter Ploegman, and Deirdre Kelly of the DRC at ada@middlebury.edu for more information. All discussions will remain confidential.

Grading

I will determine final grades according to the following percentages.

Homework	50%
Midterm Exam	20%
Final Exam	20%
Class Participation	10%

Tentative Schedule of Topics

Week Beginning	**Topics**
Sept 11	Parametrized curves, arc length
Sept 18	Frenet-Serret frame, curvature, torsion, derivative of a map $f : \mathbb{R}^n \to \mathbb{R}^m$
Sept 25	Regular surfaces
Oct 2	Tangent planes, first fundamental form, change of parameters
Oct 9	Smooth functions on surfaces, orientability and the Gauss map
Oct 16	Derivative of smooth function between surfaces, global and local isometries
Oct 23	(Midterm exam), the derivative of the Gauss map
Oct 30	The second fundamental form and normal curvature
Nov 6	Gaussian and mean curvatures, local expressions of curvatures
Nov 13	Christoffel symbols, Gauss' Theorema Egregium, geodesic curvature
Nov 20	Happy Thanksgiving!
Nov 27	Geodesic curvature, geodesics, turning of tangents
Dec 4	Gauss-Bonnet theorem: local and global
Dec 11	Review