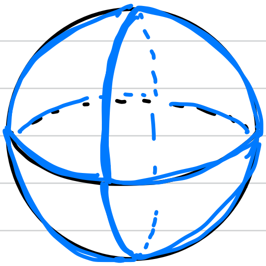
For any triangulation, we can count

$\bar{F}$ = # of faces

$\bar{E}$ = # of edges

$\bar{V}$ = # of vertices

Ex.

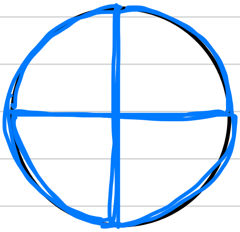


$\bar{F} = 8$

$\bar{E} = 12$

$\bar{V} = 6$

Ex.



$\bar{F} = 4$

$\bar{E} = 8$

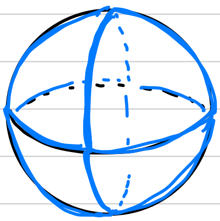
$\bar{V} = 5$

<u>Defn</u> Sps that $\mathcal{T}$ is a triangulation of a regular region $R \subset S$. The <u>Euler-Poincaré characteristic</u> of S is

often, we'll consider $R = S$.

$$\chi(R) = \bar{F} - \bar{E} + \bar{V}.$$

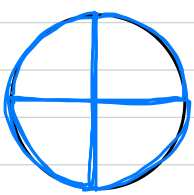
<u>Ex.</u>



With this $\mathcal{T}$,

$$\chi(\text{sphere}) = 8 - 12 + 6 = 2$$

<u>Ex.</u>



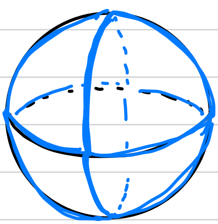
With this $\mathcal{T}$,

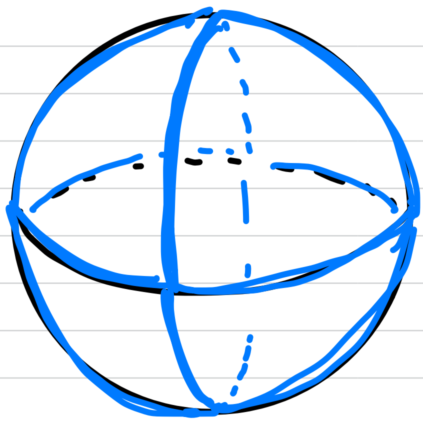
$$\chi(\text{disk}) = 4 - 8 + 5 = 1$$

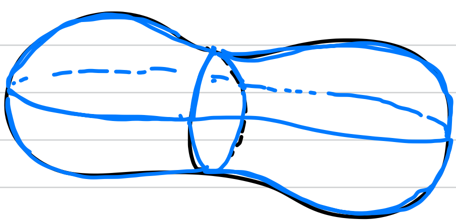
$\chi(\overset{R}{S})$ is indep of choice of $\mathcal{P}$.

We'll show: $\chi(\overset{R}{S})$ does not depend on $\mathcal{P}$.

But observe: For any $\mathcal{P}$, $\bar{F}$, $\bar{E}$, $\bar{V}$ don't depend at all on geometry, or even differentiability.







So: $\chi(S)$ is a topological invariant.