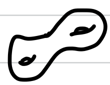
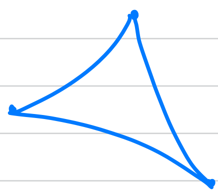
Triangulations and the Euler Characteristic

↳ a topological invariant of a surface... we'll prove this.

Sps that S is a closed and bounded orientable surface (possibly with boundary) in $\mathbb{R}^3$

e.g. sphere, disk, torus, , ...

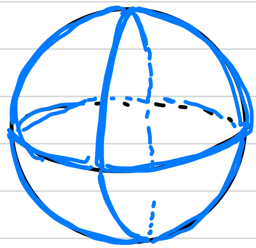
A generalized triangle on S is a simple region with three vertices and $\underbrace{\theta_0, \theta_1, \theta_2 \neq 0}_{\text{exterior angles.}}$

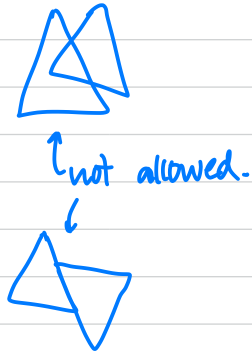


A triangulation of S (or region $R \subset S$) is a finite set $\mathcal{T}$ of triangles Δ_i on S such that

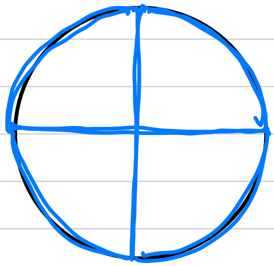
1. $S = \bigcup_i \Delta_i$
2. $\Delta_i \cap \Delta_j$ is either $\emptyset$, or a vertex, or an edge.

Ex. S = sphere





Ex. closed disk



bounded by piecewise diff'ble, regular, simple, closed curve.

Thm Every regular region of a regular surface admits a triangulation.

Thm If S is an oriented surface, $\{(\bar{x}_\alpha, U_\alpha)\}$ is a covering of S by charts, and R is a regular region in S, then there is a triangulation of R such that every triangle is contained in one of the $\bar{x}_\alpha(U_\alpha)$'s.

so: we can work locally ... use local GB thm.

triangle on left as you walk around bdry.

Thm For any triangulation of an oriented region $R \subset S$, we can orient each triangle positively relative to the orientation of S. In that case shared edges have opposite orientations.