Before the proof of the local Gauss-Bonnet theorem, three lemmas:

<u>Lemma 1</u>. If $\bar{p}$ is any pt in a regular surface S, there exists a chart $(\bar{x}, U)$ about $\bar{p}$ such that $F = 0$.

as a function on U.

proof: omitted.

<u>Lemma 2</u>. If $(\bar{x}, U)$ is a chart such that $F = 0$ as a function, then the Gaussian curvature K is given by

$$K = -\frac{1}{2\sqrt{EG}}\left(\frac{\partial}{\partial v}\left(\frac{E_v}{\sqrt{EG}}\right) + \frac{\partial}{\partial u}\left(\frac{G_u}{\sqrt{EG}}\right)\right)$$

proof: exercise.

Lemma 3 If $(\bar{x}, U)$ is a chart such that $F = 0$, then along

as a function

a curve $\bar{\alpha}_i(s) = (u(s), v(s))$

$\bar{x}^{-1} \circ \bar{\alpha}_i(s)$

ith arc of bdry curve of R

angle measure function

$$\kappa_g(s) = \frac{1}{2\sqrt{EG}}\left[G_u \frac{dv}{ds} - E_v \frac{du}{ds}\right] + \frac{d\varphi_i}{ds}$$

proof: omitted.

uses Christoffel symbols.