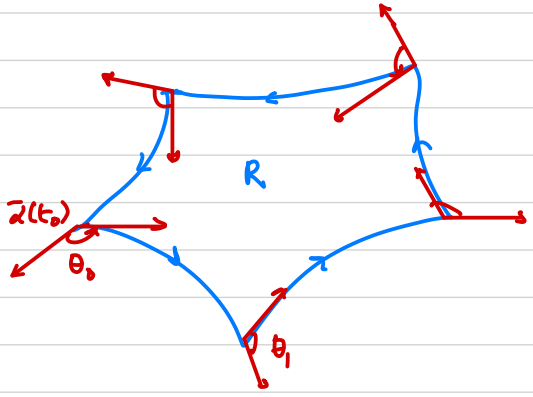
Thm (Local Gauss-Bonnet)

Sps R is a simply-connected region in a regular surface S bounded by a simple, closed, piecewise regular curve $\bar{\alpha}$ which has positive orientation relative to R. Let $\bar{\alpha}(t_0), \ldots, \bar{\alpha}(t_n)$ be the vertices of $\bar{\alpha}$, with exterior angles $\theta_0, \theta_1, \ldots, \theta_n$.



Then $$\sum_{i=0}^{n} \int_{t_i}^{t_{i+1}} \kappa_g(t)\,dt + \iint_R K\,dA + \sum_{i=0}^{n} \theta_i = 2\pi$$

κ_g: geodesic curvature; K: Gaussian curve

curves (edges) — geodesic curvature; surface (face) — Gaussian curve; points (vertices)