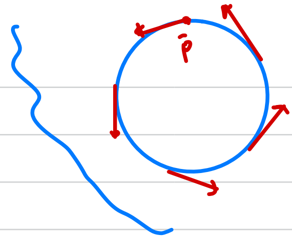
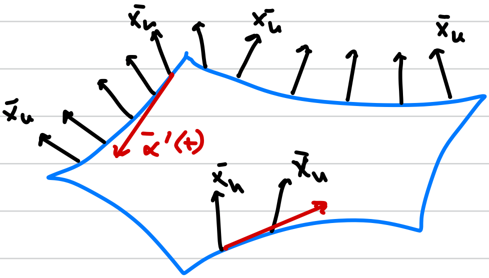
Goal: measure "turning" of tangents.



Sps S is oriented, compatible with orientation induced by $(\bar{x}, U)$

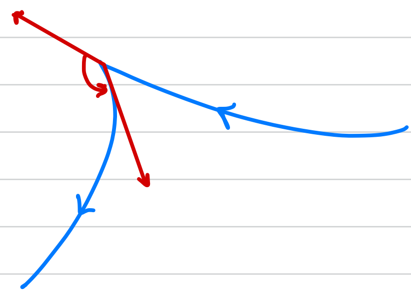


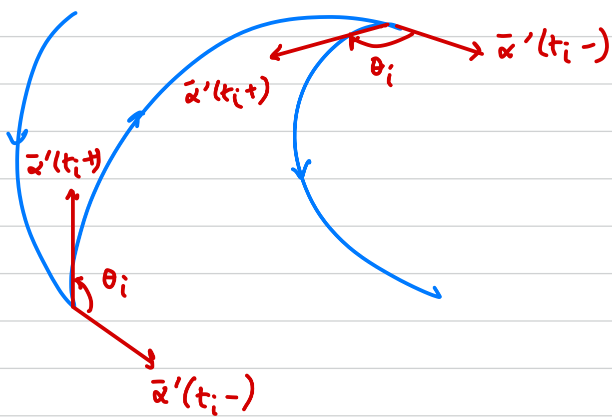
Along each arc $\bar{\alpha}\Big|_{[t_i, t_{i+1}]}$ let

$\varphi_i : [t_i, t_{i+1}] \to [0, 2\pi)$ measure angle counterclockwise of $\bar{\alpha}'(t)$ from $\bar{x}_u$, measured ccw when looking from "above" down N.

So $\varphi_i(t_{i+1}) - \varphi(t_i)$ measures turning of tangents on i^{th} arc.

At vertices, angle jumps:

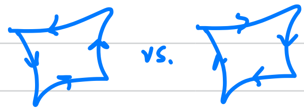




Let $-\pi < \theta_i < \pi$ be angle b/w incoming tangent $\bar{\alpha}'(t_i-)$ and $\bar{\alpha}'(t_i+)$ measured first to second, sign determined by looking from "above" down N.

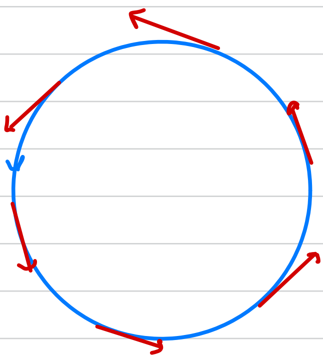
θ_i is called the external angle at $\bar{\alpha}(t_i)$

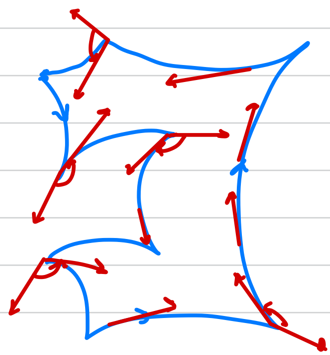
Thm (Hopf... Turning Tangents)



In the situation described above,

$$\sum_{i=0}^{n} \left(\varphi_i(t_{i+1}) - \varphi_i(t_i)\right) + \sum_{i=0}^{n} \theta_i = \pm 2\pi$$





Idea: closed, simple, piecewise regular curve is "like" a circle. Tangents move through $\pm 2\pi$ radians as you move around $\bar{\alpha}$, even if $\bar{\alpha}$ is complicated.

proof: topological, somewhat complicated.