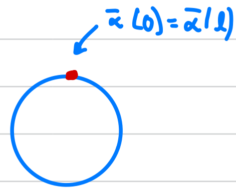
The Gauss-Bonnet Theorem - Preliminaries

↳ G-B Thm gives relationship b/w topology and geometry of a surface.

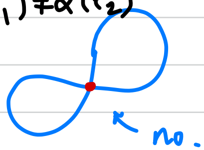
<u>Defn</u> Sps $\bar{\alpha} : [0, \ell] \to S$ is a continuous curve such that

1. $\bar{\alpha}(0) = \bar{\alpha}(\ell)$ ($\bar{\alpha}$ is closed)



2. If $t_1, t_2 \in [0, \ell)$ are such that $t_1 \neq t_2$,

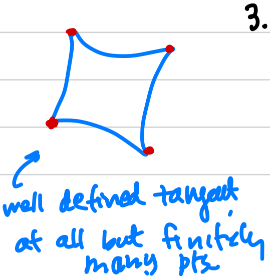
   then $\bar{\alpha}(t_1) \neq \bar{\alpha}(t_2)$ (no self-intersections)



3. there is a subdivision

$$0 = t_0 < t_1 < t_2 < \cdots < t_n < t_{n+1} = \ell$$

s.t. $\bar{\alpha}\big|_{[t_i, t_{i+1}]}$ is diff'ble and regular.

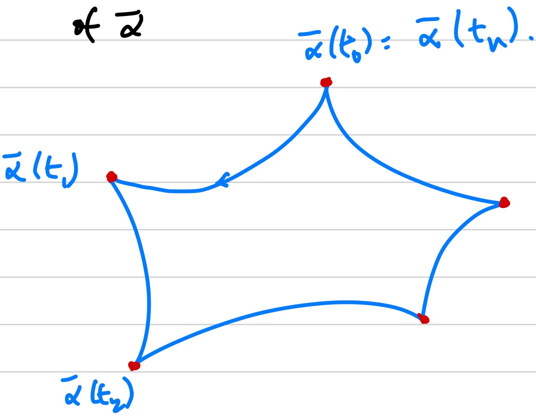


Then we say that $\bar{\alpha}$ is a

① closed, ② simple, ③ piecewise regular

parametrized curve.

The points $\bar{\alpha}(t_i)$, $i = 0, 1, \ldots, n$ are called vertices of $\bar{\alpha}$



From now on, assume image of $\bar{\alpha}$ is contained in image of chart $(\bar{x}, U)$.

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