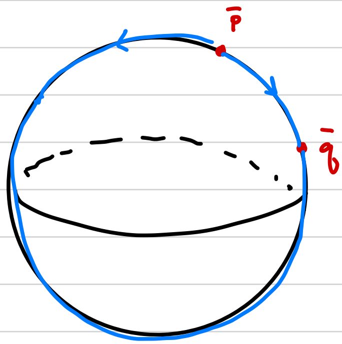
Finally, geodesics are locally length-minimizing.

↳ stationary point of length functional on curves $\bar{p}$ to $\bar{q}$.

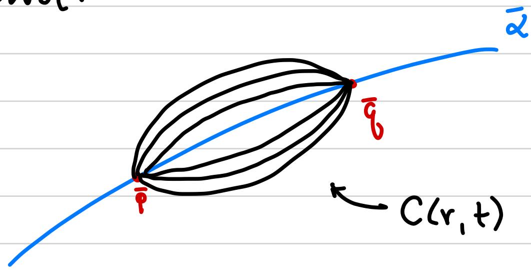


calculus of variations...

Thm Sps. $\bar{\alpha} : (-\varepsilon, \varepsilon)$ to S is a curve parametrized proportional to arc length and sps. $-\varepsilon < a < b < \varepsilon$.

Sps $\bar{\alpha}(a) = \bar{p}$ and $\bar{\alpha}(b) = \bar{q}$. If $\bar{\alpha}$ is the shortest curve joining $\bar{p}$ to $\bar{q}$, then $\bar{\alpha}$ is a geodesic on interval (a, b).

sketch of proof:



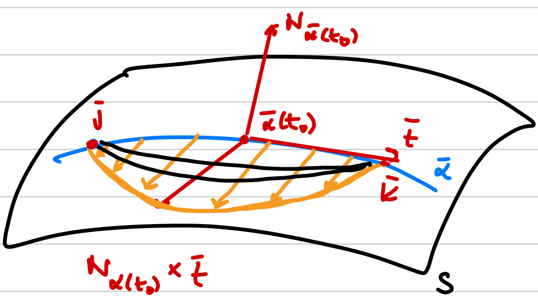
1. Construct a family of curves

$$C(r, t)$$

which curve parameter of curve

called a variation

b/w ~~$\bar{p}$~~ $\bar{\alpha}(a)$ and ~~$\bar{q}$~~ $\bar{\alpha}(b)$ with $C(0,t) = \bar{\alpha}(t)$.



* For contradiction *

⇝ Sps $\kappa_g(t_0)$ is NOT 0. * *

Then $\kappa_g(t) \neq 0$ on an interval $[j, k]$...

2. Show that assumption that $\kappa_g \neq 0$ leads to a contradiction that $\bar{\alpha}$ minimizes length:

$r = 0$

Create length functional:

$$L(\bar{r}) = \text{length of } C(r,t)$$
$$= \int_a^b \sqrt{\left(\frac{\partial C}{\partial t}(r,t) \cdot \frac{\partial C}{\partial t}(r,t)\right)}\, dt.$$

Compute $L'(r)$. Find:

$$0 = L'(0) < 0$$

assumption that $\bar{\alpha}$ minimizes length.

from construction above, based on hypothesis that: $\kappa_g \neq 0$

A contradiction. Thus: $\kappa_g(t) = 0$ for all t, so $\bar{\alpha}$ a geodesic.