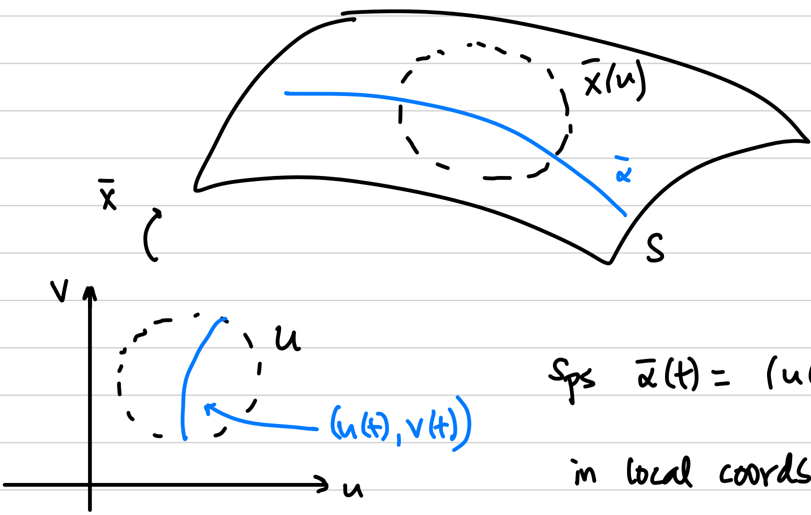
Properties of Geodesics

Sps $\bar{\alpha}$ a curve on a surface S.

Let $(\bar{x}, U)$ be a chart and sps a segment of $\bar{\alpha}(t)$ contained in $(\bar{x}, U)$.



Sps $\bar{\alpha}(t) = (u(t), v(t))$ in local coords (ie. $\bar{x}^{-1} \circ \bar{\alpha}(t) = (u(t), v(t))$)

details omitted

Can express condition of being a geodesic, i.e.

$$\frac{D\bar{\alpha}'(t)}{dt} = 0$$

Christoffel symbols

in terms of u, v, u', v', u'', v'', and Γ^k_{ij}

recall: these depend only on E, F, G, (ie. FFF)

This implies to important theorems:

<u>Thm</u> Given a curve $\bar{\alpha}$ in a surface S, the question of whether $\bar{\alpha}$ is a geodesic (ptwise or on whole curve) depends only on $\bar{\alpha}$ and S and first fundamental form, i.e. it is intrinsic.

<u>Corollary</u> If $\bar{\alpha}$ is a geodesic in S_1 and $\varphi: S_1 \to S_2$ is a local isometry, then $\varphi \circ \bar{\alpha}$ is a geodesic on S_2.

geodesics preserved by isometries.

Note: k_g is intrinsic as well.

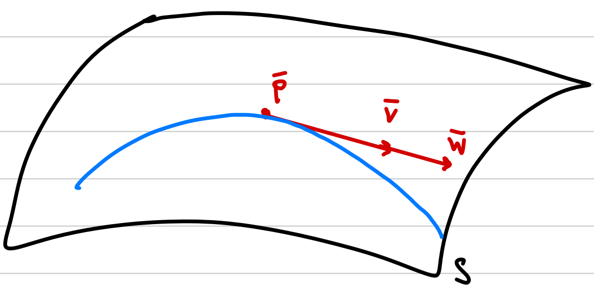
The equations used to express condition

$$\frac{D\bar{\alpha}'(t)}{dt} = \bar{0}$$ ← geodesic condition

are differential equations.

↳ Theory of differential equations says that for any initial conditions, for such a system there exists and unique solution (i.e. curve $\bar{\alpha}$) satisfying these equations.

Upshot:



<u>Thm</u> Given a point $\bar{p} \in S$ and a vector $\bar{v} \in T_p S$,

there exists a~~nd~~ unique geodesic $\bar{\alpha}$ with

$\bar{\alpha}(0) = \bar{p}$ and $\bar{\alpha}'(0) = \bar{v}$ with maximal domain (a, b)

↳ longer $\bar{x}$ $\Rightarrow$ max'l domain shorter