Note: To be a geodesic is a property both of curve C in S and of its parametrization $\bar{\alpha}$.

trace of $\bar{\alpha}$.

If $\bar{\alpha}(t)$ a geodesic,

$$\frac{D\bar{\alpha}'(t)}{dt} = \bar{0} \text{ for all } t, \quad \text{definition}$$

so $\bar{\alpha}''(t) \perp T_{\bar{\alpha}(t)} S$

thus (*) $\bar{\alpha}''(t) \cdot \bar{\alpha}'(t) = 0$ for all t. b/c $\bar{\alpha}'(t) \in T_{\bar{\alpha}(t)} S$

But $\bar{\alpha}''(t) \cdot \bar{\alpha}'(t) = \frac{1}{2} \frac{d}{dt} (\underbrace{\bar{\alpha}'(t) \cdot \bar{\alpha}'(t)}_{\text{antideriv. of } \bar{\alpha}''(t) \cdot \bar{\alpha}'(t)})$

So integrating (*) gives $\bar{\alpha}'(t) \cdot \bar{\alpha}'(t) = c$

some constant c.

Thus if $\bar{\alpha}(t)$ a geodesic, $|\bar{\alpha}'(t)| = \text{constant}$.

Note: $\tilde{\alpha}(t)$ not necessarily p.b.a.l., but

it is parametrized proportional to arc length.

↳ constant mult of