Geodesics

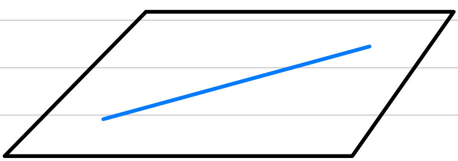
Defn A regular parametrized curve $\bar{\alpha}$ is <u>geodesic</u> at the point $\bar{\alpha}(t)$ if $\dfrac{D\bar{\alpha}'(t)}{dt} = \bar{0}$. $\rightsquigarrow$ $\bar{\alpha}''(t) \perp T_{\bar{\alpha}(t)}S$.

If $\bar{\alpha}$ is geodesic at all of its points, we say $\bar{\alpha}$ is a geodesic.

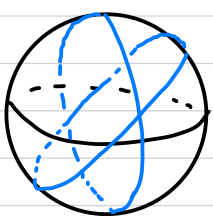
Note: to be geodesic is a pointwise property that we extend to curves.

similar to continuity or diff'bility

Ex

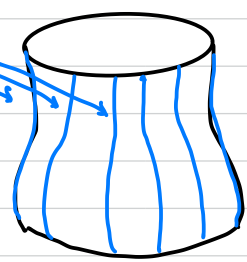


straight lines in planes



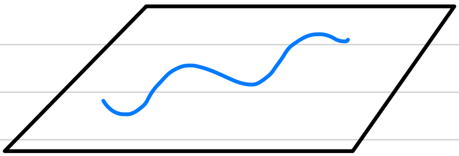
great circles (equators) in spheres

can prove these are geodesics

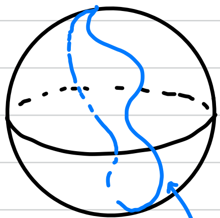


meridians in surfaces of revolution

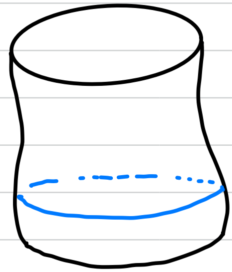
Nonex.



any nonstraight curve in plane:



$\bar{\alpha}$ "wiggles" in tangential direction



in general, parallels on surfaces of revolution are not geodesics.

Properties of geodesics:

- $\dfrac{D\bar{\alpha}'(t)}{dt} = \bar{0}$

- $\bar{\alpha}''(t) \perp T_{\bar{\alpha}(t)}S$ for all t. (ie. $\bar{\alpha}''(t) \parallel N_{\bar{\alpha}(t)}$)

- "straight" from p.o.v. of surface. No side to side, tangential wiggling

- Shortest path b/w pts. in a surface

  we'll see...