local expressions.

Christoffel Symbols

Sps S is a regular surface and that $(\bar{x}, U)$ is a chart about $p \in S$.

T_pS

Recall: $N_u = dN_p(\bar{x}_u) = a_{11}\bar{x}_u + a_{21}\bar{x}_v$

$N_v = dN_p(\bar{x}_v) = a_{12}\bar{x}_u + a_{22}\bar{x}_v$

local expressions of II_p

In calculating e, f, g, we used $\bar{x}_{uu}, \bar{x}_{uv}, \bar{x}_{vv}$

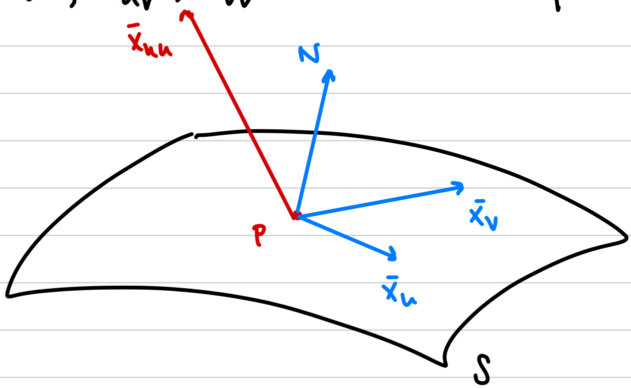
e.g.:

$e = -dN_p(\bar{x}_u)\cdot\bar{x}_u$

$= -N_u \cdot \bar{x}_u$

$= N \cdot \bar{x}_{uu}$

Note: $\bar{x}_{uu}, \bar{x}_{uv}, \bar{x}_{vv}$ not nec. in T_pS:



$T_p\mathbb{R}^3$

At $\bar{p} \in S \subset \mathbb{R}^3$, $\{\bar{x}_u, \bar{x}_v, N\}$ form a basis for a copy of $\mathbb{R}^3$ based at $\bar{p}$. Write $\bar{x}_{uu}, \bar{x}_{uv}, \bar{x}_{vv}$ in terms of this:

Gamma

$u=1$

$v=2$

$$\bar{x}_{uu} = \Gamma^1_{11}\bar{x}_u + \Gamma^2_{11}\bar{x}_v + L_1 N$$

$$\bar{x}_{uv} = \Gamma^1_{12}\bar{x}_u + \Gamma^2_{12}\bar{x}_v + L_2 N$$

equal

$$\bar{x}_{vu} = \Gamma^1_{21}\bar{x}_u + \Gamma^2_{21}\bar{x}_v + \bar{L}_2 N$$

$$\bar{x}_{vv} = \Gamma^1_{22}\bar{x}_u + \Gamma^2_{22}\bar{x}_v + L_3 N$$

↳ sometimes L_1, L_2, L_3 called L, M, N.

Γ_{ij}^{k}'s are called <u>Christoffel symbols</u>.

unique expression of vectors

Since $\{\bar{x}_u, \bar{x}_v, N\}$ is a basis and $\bar{x}_{uv} = \bar{x}_{vu}$,

we have:

$$\Gamma_{12}^{1} = \Gamma_{21}^{1}, \quad \Gamma_{12}^{2} = \Gamma_{21}^{2}, \quad L_2 = \bar{L}_2$$

earlier

Also: $e = N \cdot \bar{x}_{uu} = L_1$ $\quad (N \perp \bar{x}_u,\ N \perp \bar{x}_v,\ N \cdot N = 1)$

$f = N \cdot \bar{x}_{uv} = L_2 = \bar{L}_2$

$g = N \cdot \bar{x}_{vv} = L_3$

all referred to sometimes as L, M, N