Remarkable Theorem

Christoffel Symbols and Gauss' Theorema Egregium

Recall: arc length, angles, area, distance b/w pts in S

all defined using $I_p(\bar{v}) = (\bar{v} \cdot \bar{v})$.

↳ depend only on inner product on T_pS

↳ they are intrinsic to the surface.

OTOH, Gaussian curvature, mean curvature, principal curvatures

defined using $II_p(\bar{v}) = -(dN_p(\bar{v}) \cdot \bar{v})$

↳ Need that S sits in $\mathbb{R}^3$ to determine N.

Goal: Show Gaussian curvature K is intrinsic; depends only on surface, not on how it sits in $\mathbb{R}^3$.

↳ Need to show: K depends only on I_p.

(this is Gauss' Theorema Egregium