Gaussian and Mean Curvature

Let $\bar{p} \in S$ and let $dN_p : T_pS \to T_pS$. The <u>Gaussian curvature</u> K at $\bar{p}$ is:

κ_1, κ_2 $\parallel$ $\vee$ k_1 k_2 throughout

$$K = k_1 k_2 = \text{product of principle curvatures}$$

$$= (-\lambda_1)(-\lambda_2)$$

doesn't dep on orientation.

eigenvalues of dN_p

$$= \det(dN_p).$$

The <u>mean curvature</u> H at $\bar{p}$ is:

$$\frac{k_1 + k_2}{2} = \text{average} = -\frac{1}{2}\operatorname{tr} dN_p$$

$dN_p = \begin{bmatrix} -k_1 & \\ & -k_2 \end{bmatrix}$

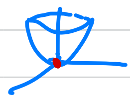
rel. to eigenvector basis

Defn $\bar{p} \in S$ is called

1. elliptic if $K > 0$ ~ (k_1, k_2 same sign) (sphere)
2. hyperbolic if $K < 0$ ~ (k_1, k_2 opp sign) (saddle)
3. parabolic if $K = 0$ but one of $k_1, k_2 \neq 0$ (cylinder)
4. planar if $K = 0$ and $k_1 = k_2 = 0$ (plane)

Defn If at $\bar{p} \in S$ $k_1 = k_2$, $\bar{p}$ is called an umbilic point.

↳ ex: any pt on plane, any pt on sphere, $(0,0,0)$ on paraboloid $z = x^2 + y^2$



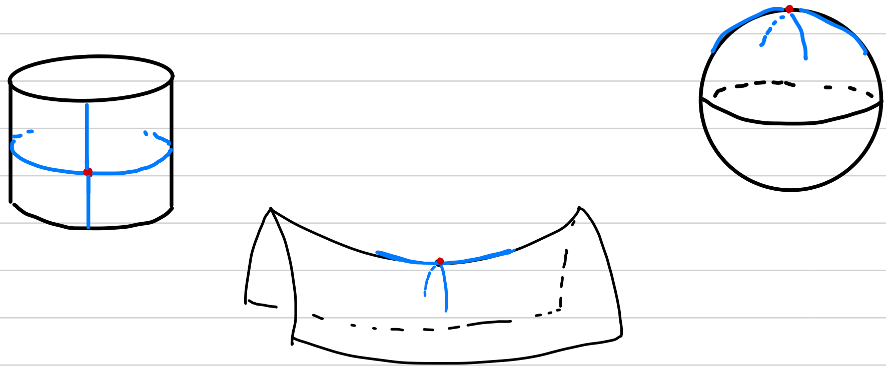
Defn A curve $\bar{\alpha}$ is called a line of curvature if

$$dN_p(\bar{\alpha}'(t)) = \lambda(t)\bar{\alpha}'(t)$$

↳ $\bar{\alpha}'(t)$ an eigenvector for all t

equiv: $\bar{\alpha}'(t)$ is a principle direction for all t.

<u>Ex.</u>



<u>Defn</u> For $\bar{p} \in S$ an <u>asymptotic direction</u> of S at $\bar{p}$ is a direction in T_pS for which $\kappa_n = 0$

equiv. $(dN_p(\bar{w}) \cdot \bar{w}) = 0$

An <u>asymptotic curve</u> is a curve $\bar{\alpha}(t)$ for which $\bar{\alpha}'(t)$ is an asymptotic direction for all $\bar{\alpha}'(t)$.

<u>Ex</u>