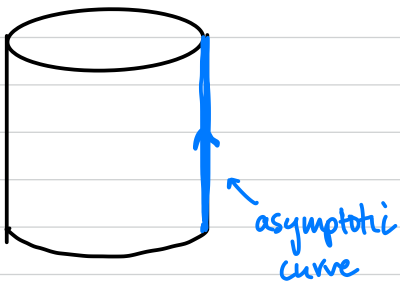
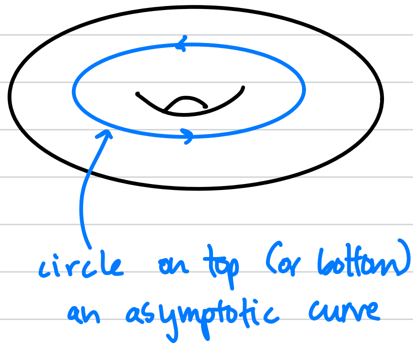
Defn An **asymptotic curve** in S is a regular, connected curve C given by $\bar{\alpha}(t)$ such that $\bar{\alpha}'(t)$ is an asymptotic (zero normal curvature) direction at $\bar{\alpha}(t)$ for all t.

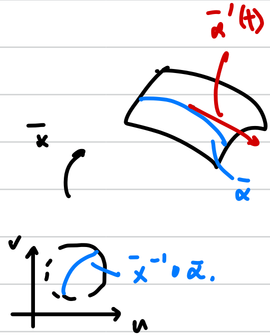
Ex





Note: If $\bar{\alpha}(t)$ is an asymptotic curve,

$$\mathrm{II}_{\alpha(t)}(\bar{\alpha}'(t)) = 0 \quad \text{for all } t.$$



In local coords, if $\bar{x}^{-1} \circ \bar{\alpha}(t) = (u(t), v(t))$ then

$$\bar{\alpha}'(t) = u'(t)\bar{x}_u + v'(t)\bar{x}_v .$$

So:

$$0 = \mathrm{II}_{\bar{\alpha}(t)}(\bar{\alpha}'(t))$$

$$= e(u(t), v(t))(u'(t))^2 + 2f(u(t), v(t))u'(t)v'(t) + g(u(t), v(t))(v'(t))^2$$

$$= e(u')^2 + fu'v' + g(v')^2 \; *$$ (all derivs. w.r.t parameter t)

So a curve $\bar{\alpha}(t)$ is asymptotic $\Leftrightarrow$ $(\bar{x}^{-1} \circ \bar{\alpha})(t)$ satisfies $*$ for all t.