Ex. Sps $e = f = g = 0$.

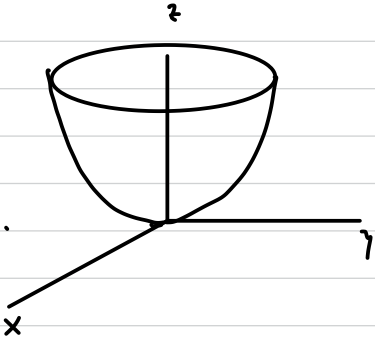
Then
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \frac{-1}{EG-F^2}\begin{bmatrix} G & -F \\ -F & E \end{bmatrix}\begin{bmatrix} e & f \\ f & g \end{bmatrix}$$

$$K_p = \frac{eg-f^2}{EG-F^2}$$

local expression of dN_p

So $\bar{p}$ is : planar.

here, 0-matrix



Ex. Paraboloid $z = x^2+y^2$

↳ chart$(\bar{x}, U) = (u, v, u^2+v^2)$.

From earlier:

$E = 1+4u^2$ $\qquad e = \dfrac{2}{\sqrt{4u^2+4v^2+1}}$

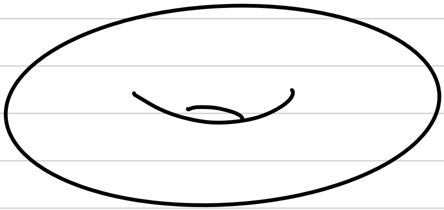
$F = 4uv$ $\qquad f = 0$

$G = 1+4v^2$ $\qquad g = \dfrac{2}{\sqrt{4u^2+4v^2+1}}$

So: $$K = \frac{eg-f^2}{EG-F^2} = \frac{\frac{4}{4u^2+4v^2+1}}{4u^2+4v^2+1} = \frac{4}{(4u^2+4v^2+1)^2}$$

So Gaussian curvature is pos. everywhere, decreases as u^2+v^2 grows.— i.e. as you move up paraboloid.

Ex. Torus



surface of revolution

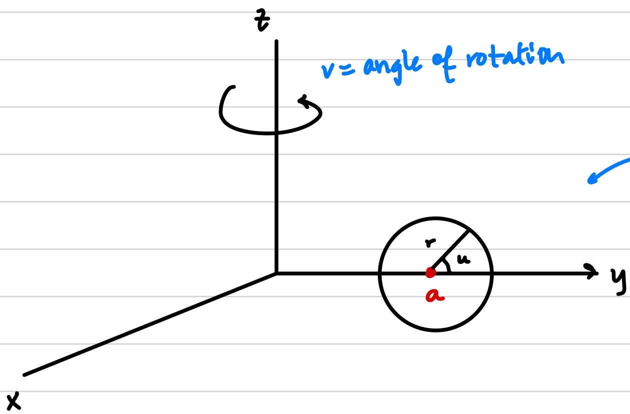
Chart: $\bar{x}(u,v) = \left((a + r\cos u)\cos v,\ (a + r\overset{\cos u}{\cancel{\sin u}})\sin v,\ r\sin u\right)$

$a > r > 0$, both fixed

$0 < u, v < 2\pi$

v = angle of rotation

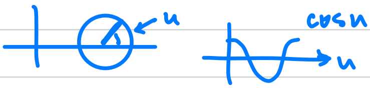
$(a,0) + (r\cos u, r\sin u)$



Find :

$E = r^2 \qquad F = 0 \qquad G = (a + r\cos u)^2$

$e = r \qquad f = 0 \qquad g = \cos u\,(a + r\cos u)$



Note: $EG - F^2 > 0$
↳ denom > 0.

So $K = \dfrac{eg - f^2}{EG - F^2} = \dfrac{\cos u}{r(a + r\cos u)}$

So sign of K determined by sign of cos u

$u = \frac{\pi}{2}, \frac{3\pi}{2}$ (top/bottom of torus) → $K = 0$

$\frac{\pi}{2} < u < \frac{3\pi}{2}$ (inside torus) → $K < 0$

$u < \frac{\pi}{2}$ or $u > \frac{3\pi}{2}$ (outside torus) → $K > 0$