Now: a formula for K, easy to compute:

$\swarrow$ Gaussian curvature at p

Recall: $K_p = \kappa_1 \kappa_2$ $\leftarrow$ principle curvatures.

$= (-\lambda_1)(-\lambda_2)$ $\leftarrow$ eigenvalues of dN_p

dN_p relative to eigenbasis is $\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}$

$= \det dN_p$ $\leftarrow$ indep of choice of basis.

local expression of dN_p relative to chart $(\bar{x}, U)$

So, relative to $\{\bar{x}_u, \bar{x}_v\}$ matrix of dN_p is: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

so $K_p = a_{11} a_{22} - a_{12} a_{21}$.

$\uparrow \det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

But from before: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \dfrac{-1}{EG-F^2} \begin{bmatrix} G & -F \\ -F & E \end{bmatrix} \begin{bmatrix} e & f \\ f & g \end{bmatrix}$

nxn matrix

Recall: $\det(AB) = \det A \det B$ and $\det(cA) = c^n \det A$.

So $K_p = \det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \dfrac{eg - f^2}{EG - F^2}$ $\Big\}$ * local expression of Gaussian curvature. (Easy to compute)

Similarly $H = \frac{1}{2}(\kappa_1 + \kappa_2)$ ← principle curvatures

$= -\frac{1}{2}(\lambda_1 + \lambda_2)$ ← eigenvalues of dN_p

$= -\frac{1}{2} \operatorname{tr} (\lambda_1 + \lambda_2)$ ← independent of basis

So $H = -\frac{1}{2}(a_{11} + a_{22})$ $\Big\}$ * local expression of mean curvature