multivariable: $f: \mathbb{R}^2 \to \mathbb{R}$

$\frac{\partial f}{\partial x}$ or f_x

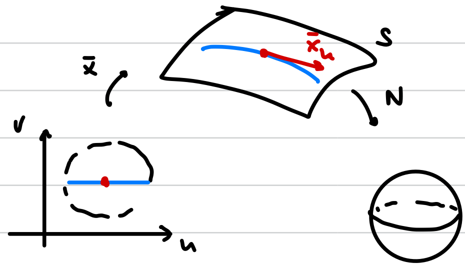
Notation!

change of N in u-direction

Let $N_u = dN_p(\bar{x}_u)$

$N_v = dN_p(\bar{x}_v)$

change of N in v-direction



So
$$e(u,v) = -(N_u \cdot \bar{x}_u)$$
$$f(u,v) = -(N_u \cdot x_v) = -(N_v \cdot \bar{x}_u)$$
$$g(u,v) = -(N_v \cdot \bar{x}_v)$$

Note: for computational purposes, N_u and N_v can be a pain to compute. But since $(N \cdot \bar{x}_u) = 0 = (N \cdot \bar{x}_v)$ always, we have

$$\frac{d}{du}(N \cdot \bar{x}_u) = 0$$

$$\Rightarrow \underbrace{(N_u \cdot \bar{x}_u)}_{-e} + (N \cdot \bar{x}_{uu}) = 0$$

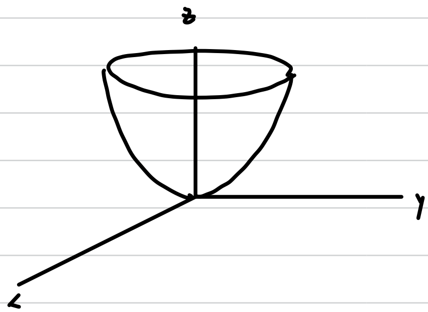
So $e = (N \cdot \bar{x}_{uu})$

Similarly, $f = (N \cdot \bar{x}_{uv}) = (N \cdot \bar{x}_{vu})$

$g = (N \cdot \bar{x}_{vv})$

easy to compute, like E, F, G.

<u>Ex</u> Paraboloid $z = x^2 + y^2$



$\bar{x}(u,v) = (u, v, u^2+v^2)$

$\bar{x}_u = (1, 0, 2u)$

$\bar{x}_v = (0, 1, 2v)$

$\bar{x}_{uu} = (0, 0, 2)$

$\bar{x}_{uv} = (0, 0, 0)$

$\bar{x}_{vv} = (0, 0, 2)$

$$N(u,v) = \frac{\bar{x}_u \times \bar{x}_v}{|\bar{x}_u \times \bar{x}_v|} = \frac{(-2u, -2v, 1)}{\sqrt{4u^2+4v^2+1}}$$

So:

$e = (N \cdot \bar{x}_{uu}) = \frac{2}{\sqrt{4u^2+4v^2+1}}$

$f = 0$

$g = (N \cdot \bar{x}_{vv}) = \frac{2}{\sqrt{4u^2+4v^2+1}}$

$E = 1 + 4u^2 \quad F = 4uv$

$G = 1 + 4v^2$