The Gauss Map, II_p, and Curvature in Local Coords.

gives a basis $\{\bar{x}_u, \bar{x}_v\}$ of T_pS.

Recall: Given a chart $(\bar{x}, U)$ about $\bar{p} \in S$, then

$I_p(\bar{x}_u)$

$$E(u,v) = (\bar{x}_u \cdot \bar{x}_u)$$

$$F(u,v) = (\bar{x}_u \cdot \bar{x}_v)$$

$$G(u,v) = (\bar{x}_v \cdot \bar{x}_v)$$

knowing these equiv to knowing FFF.

for all $\bar{p} \in \bar{x}(U)$.

and by linearity, E, F, G completely determine $(\cdot)_p$,

i.e. $I_p(\ )$.

a.k.a. L, M, N

Goal: Find similar functions e, f, g for II_p...

use them to calculate curvature at $\bar{p} = \bar{x}(u,v)$.

Assume all parametrizations are consistent w/ orientation of S.

i.e. $$\frac{\bar{x}_u \times \bar{x}_v}{|\bar{x}_u \times \bar{x}_v|} = +N_p$$

$\{\bar{x}_u, \bar{x}_v\}$ a basis for T_pS.

Now, suppose $\bar{w} = a\bar{x}_u + b\bar{x}_v \in T_pS$, and $\bar{p} = \bar{x}(u,v)$. Then

linear map

$$\mathrm{II}_p(\bar{w}) = -\left(dN_p(a\bar{x}_u + b\bar{x}_v) \cdot a\bar{x}_u + b\bar{x}_v \right)_p$$

dN_p self-adjoint

$$= -a^2 (dN_p(\bar{x}_u)\cdot\bar{x}_u)_p - 2ab\,(dN_p(\bar{x}_u)\cdot\bar{x}_v)_p - b^2(dN_p(\bar{x}_v)\cdot\bar{x}_v)_p$$

$$= +a^2 e(u,v) + 2ab f(u,v) + b^2 g(u,v)$$

where

$$e(u,v) = -(dN_p(\bar{x}_u)\cdot\bar{x}_u)$$

self-adjoint

$$f(u,v) = -(dN_p(\bar{x}_u)\cdot\bar{x}_v) = -(dN_p(\bar{x}_v)\cdot\bar{x}_u)$$

$$g(u,v) = -(dN_p(\bar{x}_v)\cdot\bar{x}_v)$$

* local expression of II_p