$$\mathrm{II}_p(\bar{w}) = -\left(dN_p(\bar{w})\cdot\bar{w}\right)_p$$

Now, $\mathrm{II}_p(\bar{w})$:

p.b.a.l.

Sps. $\bar{w}\in T_pS$ is a unit vector and let $\bar{\alpha}(s)\in S$ with $\bar{\alpha}(0)=\bar{p}$ and $\bar{\alpha}'(0)=\bar{w}$.

Notice that for all s, $\left(N(\bar{\alpha}(s))\cdot\bar{\alpha}'(s)\right)_{\bar{\alpha}(s)}=0$ constant.

so $$\left.\frac{d}{ds}\right|_{s=0}\left(N(\alpha(s))\cdot\bar{\alpha}'(s)\right)_{\bar{\alpha}(s)}=0$$

ie. $$\Big(\underbrace{(N\circ\alpha)'(0)}_{dN_p(\bar{\alpha}'(0))}\cdot\bar{\alpha}'(0)\Big)_p+\Big(\underbrace{N(\bar{\alpha}(0))}_{N_p}\cdot\underbrace{\bar{\alpha}''(0)}_{\kappa\bar{n}(0)}\Big)_p=0$$

Q: what does this measure?

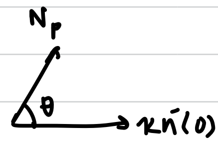
Thus: $\quad \mathrm{II}_p(\bar{w}) = \mathrm{II}_p(\bar{\alpha}'(0)) \qquad \bar{w} = \bar{\alpha}'(0)$

$= -\left(dN_p(\bar{\alpha}'(0)), \bar{\alpha}'(0)\right)_p \qquad$ defn of II_p

$= \left(N(\bar{\alpha}(0)) \cdot \alpha''(0)\right)_p \qquad$ by above

$= \left(N_p \cdot \kappa\bar{n}(0)\right)_p$

$= |N_p|\,|\kappa n(0)| \cos\theta$



$= \kappa \cos\theta$

$\kappa > 0$

$= \kappa_n \rightsquigarrow$ normal curvature of $\bar{\alpha}$.

So : $\boxed{\mathrm{II}_p(\bar{w}) = \text{normal curvature of any unit speed curve passing through } \bar{p} \text{ with tangent vector } \bar{w}.}$

Note: this implies normal curvature of curve on surface at a point $\bar{p}$ depends only on direction of curve, not on curve itself.