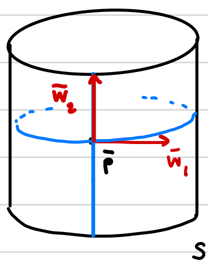
$$\mathrm{II}(\bar{w}) = -\langle dN_p(\bar{w}), \bar{w}\rangle_{\bar{p}}$$

Q: If $\bar{w}$ is a unit vector in T_pS, what does $\mathrm{II}_p(\bar{w})$ measure?

We'll see: it gives curvature of curve of intersection of S and plane defined by N_p and $\bar{w}$.

Sps. $C = \bar{\alpha}(s)$ is a unit speed curve in S w/ $\bar{\alpha}(0) = \bar{p}$.



Consider the Frenet-Serret frame $\{\bar{t}, \bar{n}, \bar{b}\}$ along $\bar{\alpha}$.

Recall: $\bar{t}(0) = \bar{\alpha}'(0)$

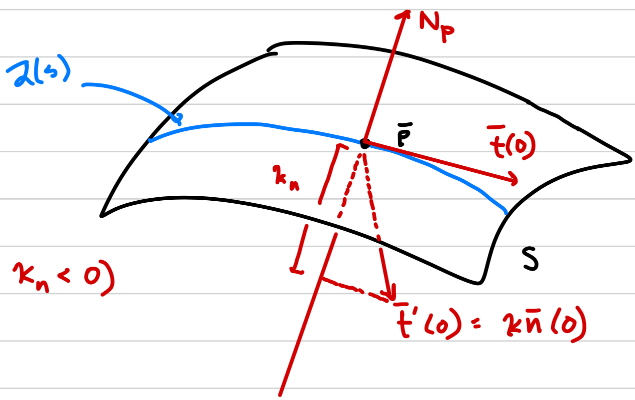
$t'(0) = \kappa \bar{n}(0) \quad (\kappa \geq 0)$

$\bar{b}(0) = t(0) \times \bar{n}(0)$

Note: $\bar{n}(0)$ may or may not lie in T_pS (so $\bar{b}(0)$ may or may not be $\parallel N(p)$).

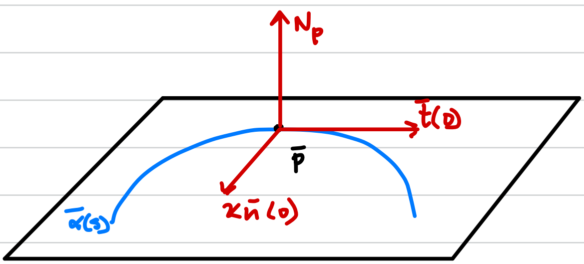
a new variable.... distinct from κ

<u>Defn</u> The <u>normal curvature</u> κ_n of C at $\bar{p}$ is the signed length of the projection of $t'(0)$ onto N_p.



(here: $\kappa_n < 0$)

<u>Ex</u> plane curve

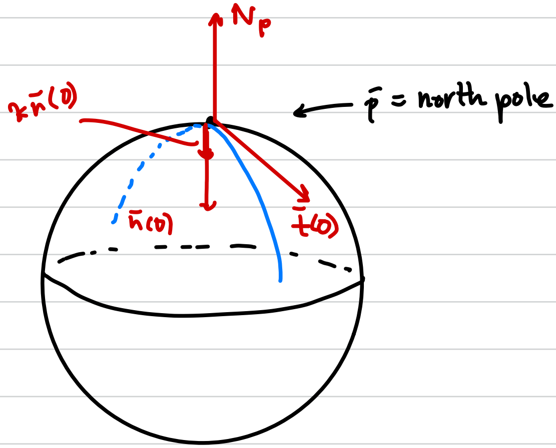


Here: $\kappa\bar{n}(0) \perp N_p$

so $\kappa_n = 0$

Ex sphere, radius 5, oriented outwards.

Sps. $\bar{\alpha}$ great circle, p.b.a.l.



Then $\bar{n}(0) = (0,0,-1)$ and $\kappa n(0) = (0,0,-\frac{1}{5})$

$\kappa = 1/5$

Here: $\kappa\bar{n}(0) \parallel N_p$

so $\kappa_n = -\frac{1}{5}$.

These two examples represent the extreme cases:

$$t'(0) \perp N_p \quad \text{or} \quad t'(0) \parallel N_p$$

$\kappa \bar{n}(0)$

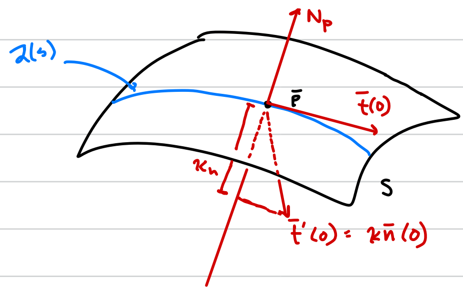
In general, sps θ = angle b/w $\bar{n}(0)$ and N_p.

On one hand: $(N_p \cdot \kappa\bar{n}(0))_p = |N_p||\kappa\bar{n}(0)|\cos\theta$ (recall $\kappa \geq 0$)

$$= \kappa \cos\theta$$

OTOH: $\cos\theta = \dfrac{\overset{\text{adj.}}{\kappa_n}}{\underset{\text{hyp.}}{|\kappa\bar{n}(0)|}}$

$$= \frac{\kappa_n}{\kappa}$$



So $\kappa_n = \kappa\cos\theta$.

Conclusion: $\kappa_n = (N_p \cdot \kappa\bar{n}(0))_p$