The Second Fundamental Form

↳ and normal curvature

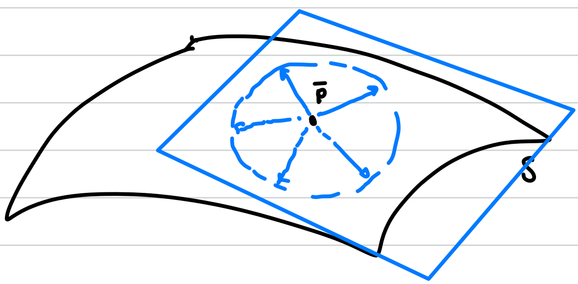
Define the <u>second fundamental form</u>

$$\mathrm{II}_p : T_pS \to \mathbb{R}$$

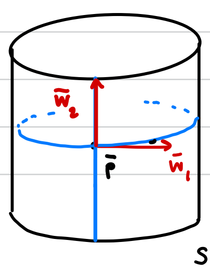
by $$\mathrm{II}_p(\bar{w}) = -\left(dN_p(\bar{w}) \cdot \bar{w}\right)_p$$

↖ recall discussion of self-adjoint $A : V \to V$ and map $\bar{v} \mapsto \langle A\bar{v}, \bar{v} \rangle$.

If we restrict $\mathrm{II}_p(\bar{w})$ to a unit circle in T_pS, it will have max/min values in direction of eigenvectors of dN_p.



Recall cylinder example:



$$\left\{ \begin{array}{l} dN_p(\bar{w}_1) = \bar{w}_1 \\ dN_p(\bar{w}_2) = 0\,\bar{w}_2. \end{array} \right.$$

$\mathrm{II}_p(\bar{w})$ max/min in direction of $\bar{w}_1$, $\bar{w}_2$.