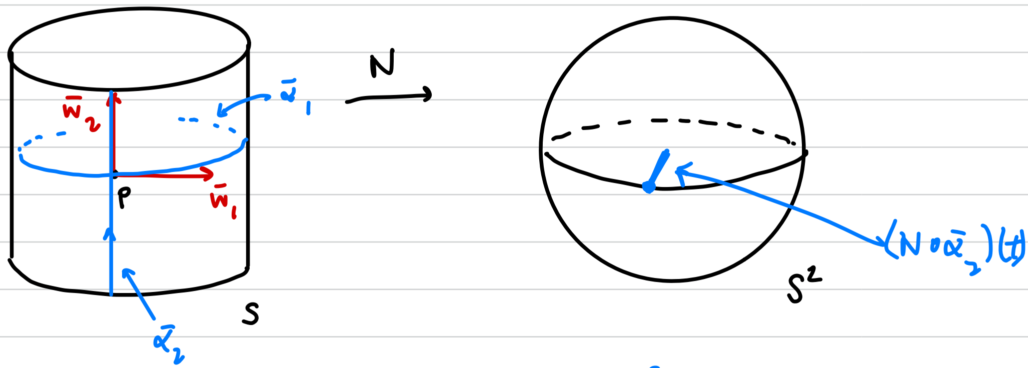
Ex. S = unit cylinder $\quad \bar p = (1,0,0) \quad \bar w_1 = (0,1,0) \in T_pS$



Let $\bar\alpha_1(t) = (\cos t, \sin t, 0)$

so $\bar\alpha(t) \in S$
$\bar\alpha(0) = \bar p$
$\bar\alpha'(0) = \bar w_1$

$$dN_p(\bar w_1) = (N\circ\bar\alpha)'(0)$$

$$= \bar\alpha'(0) = (0,1,0) = \bar w_1$$

for this particular S and $\bar\alpha$.

So in this case $dN_p(\bar w_1) = \bar w_1$.

$\bar w_1$ an eigenvector, eigenvalue 1.

Now, let $\bar w_2 = (0,0,1)$.

Let $\bar\alpha_2(t) = (1, 0, t)$

so $\bar\alpha(t) \in S$
$\bar\alpha(0) = \bar p$
$\bar\alpha'(0) = \bar w_2$

Then $dN_p(\bar w_2) = \bar 0$

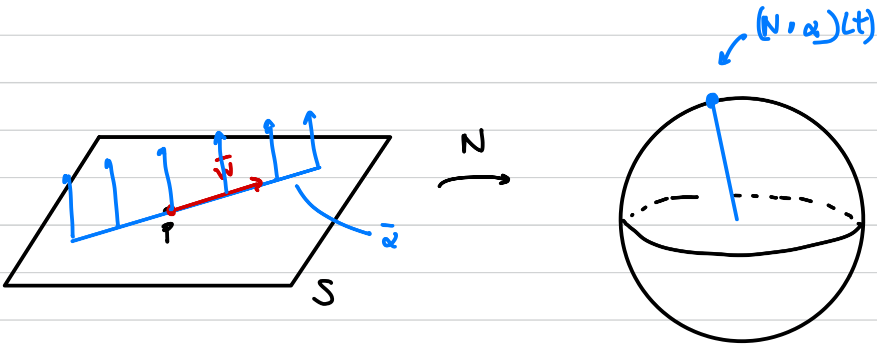
$\bar w_2$ eigenvector, eigenvalue 0.

so in this case $dN_p(\bar w_2) = \bar 0 = 0\,\bar w_2$

Note: in terms of basis $\{\bar{w}_1, \bar{w}_2\}$ of T_pS, the matrix of dN_p is:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$ ← eigenvalues 0 and 1.

<u>Ex</u>. S = plane $\bar{w}$ = any vector in T_pS.

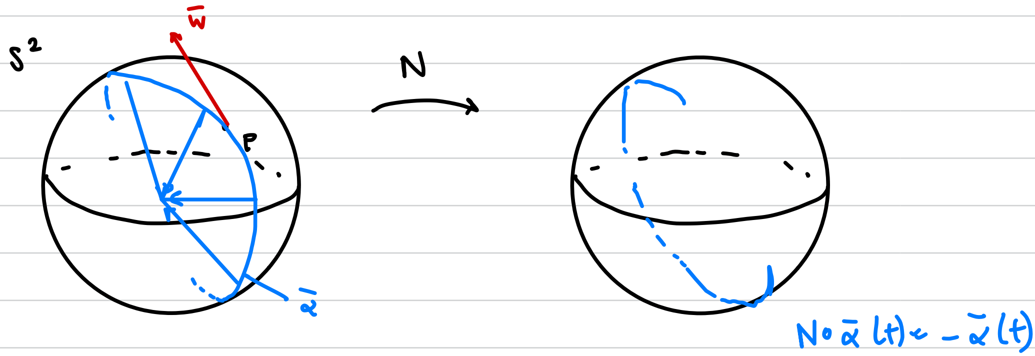


$$dN_p(\bar{w}) = \bar{0} \overset{= 0\bar{w}}{} \quad \text{for } \underline{\text{all}}\ \bar{w} \in T_pS$$

Note: matrix of dN_p relative to any basis of T_pS is:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ ← eigenvalues: 0 and 0

Ex. Unit sphere S^2, oriented inwards. Any $\bar{p} \in S^2$ and any $\bar{w} \in T_pS^2$.



At any point (x, y, z) on sphere

$$N(p) = N(x, y, z) = (-x, -y, -z)$$

For a curve $\bar{\alpha}(t)$ in sphere w/ $\bar{\alpha}(0) = \bar{p}$ and $\bar{\alpha}'(0) = \bar{w}$,

$$(N \circ \alpha)'(t) = \frac{d}{dt} N\big(\underbrace{x(t), y(t), z(t)}_{\alpha(t)}\big) = \frac{d}{dt}\big(-x(t), -y(t), -z(t)\big) = -\bar{\alpha}'(t)$$

So $(N \circ \alpha)'(0) = (-x'(0), -y'(0), -z'(0))$

$$= -\bar{\alpha}'(0)$$

$$= -\bar{w}$$

Thus $dN_p(\bar{w}) = -\bar{w}$ for all $\bar{p} \in S^2$ and $\bar{w} \in T_pS^2$.

Note: matrix of dN_p in terms of any basis of T_pS^2 is:

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$ eigenvalues: -1 and -1.

Note: If we had oriented sphere w/ outward normals, would get

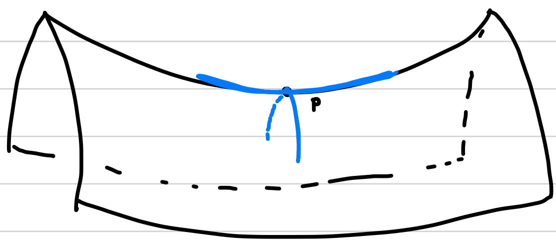
$$dN_p(\bar{w}) = \bar{w}$$

so $dN_p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ eigenvalues: 1 and 1

Ex. (sketch)

$S =$ graph of $f(x,y) = y^2 - x^2$

↳ $\{(x,y,z) \mid z = y^2 - x^2\}$ $\qquad \bar{p} = (0,0,0)$



↳ can compute dN_p has two eigenvalues, one positive, one negative.

We've seen 4 situations for dN_p:

- zero Gaussian curvature • both eigenvalues 0 : plane (planar)
- zero Gaussian curvature • one eigenvalue 0 e.g. cylinder (parabolic)
- positive Gaussian curvature • both eigenvalues $\neq 0$, same sign e.g. sphere (elliptic)
- negative Gaussian curvature • both eigenvalues $\neq 0$, opp. sign e.g. saddle (hyperbolic)

↳ will explore more carefully...