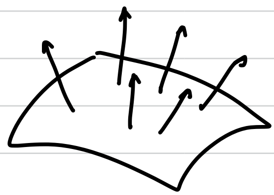
The Derivative of the Gauss Map

Suppose S is an oriented surface.



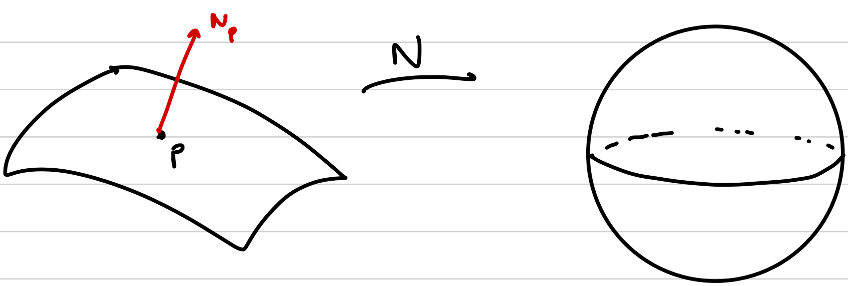
Recall: the Gauss map

$$N : S \to S^2$$

unit sphere

$$N(p) = N_p$$

place tip on S^2.

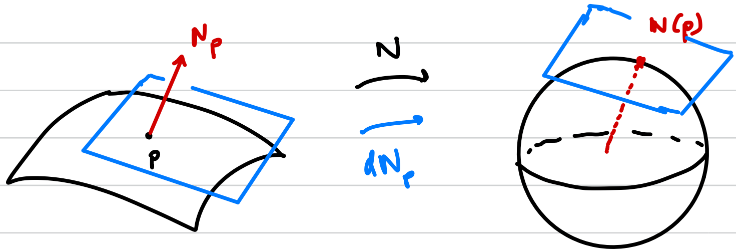


We know N is a differentiable map $S \to S^2$

so.... we can consider dN_p.

Now: $dN_p : T_p S \to T_{N(p)} S^2$ (measures change in N at p)

BUT :



$T_p S$ is parallel to $T_{N(p)} S^2$.

So we can identify $T_p S$ with $T_{N(p)} S^2$.

(i.e. think of them as the same.)

$T: V \to V$

$T\bar{v} = \lambda \bar{v}$

Thus, we can think of dN_p as

Big idea... can consider things like eigenvectors/eigenvalues...

$$dN_p : T_p S \to T_p S.$$

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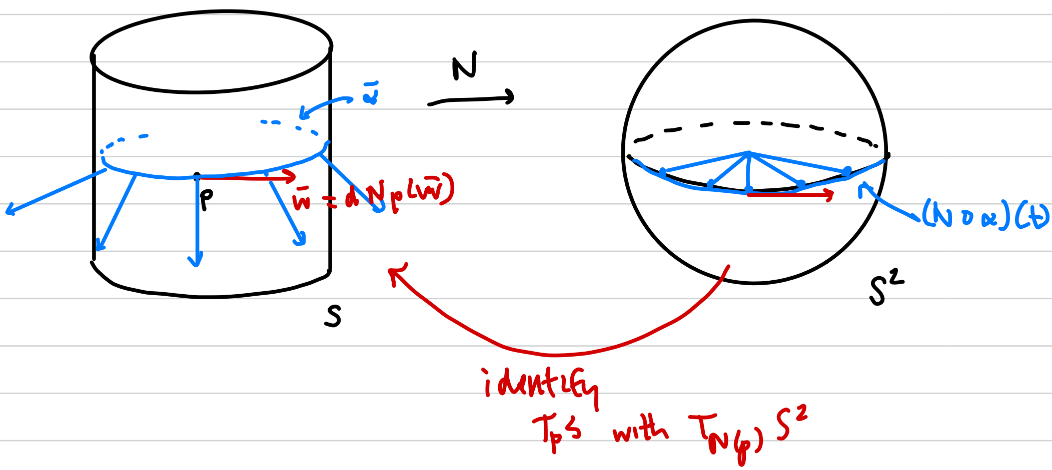
We get a lot of geometric info from dN_p, namely curvature.

To begin:

Let $\bar{w} \in T_pS$ s.t. $\bar{w} = \bar{\alpha}'(0)$ with $\bar{\alpha}(t) \in S$ and $\bar{\alpha}(0) = \bar{p}$

Consider $N \circ \bar{\alpha}(t)$



Then, by defn, $dN_p(\bar{w}) = \bar{w}$

dN_p measures change in normals as you move along $\bar{\alpha}$,

ie measures how much normals pull away from N_p normal at p

as you move in direction of $\bar{\alpha}$.