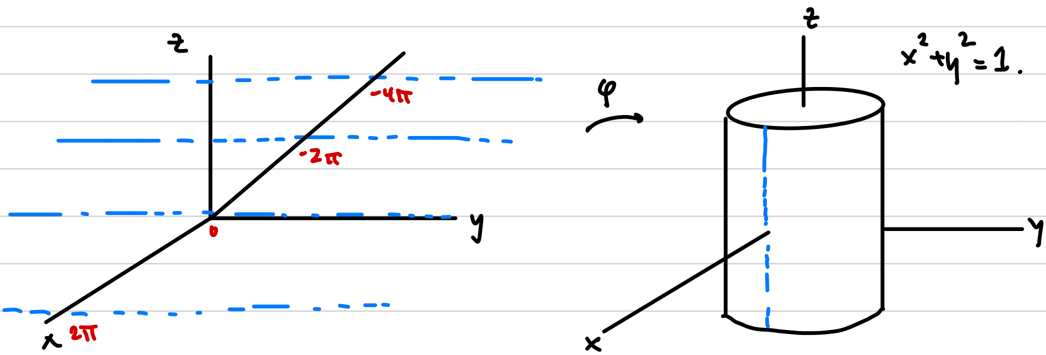
Defn A map $f: S_1 \to S_2$ is called a <u>local diffeomorphism</u> at $\bar{p}$ if there exists an open set $V \subset S_1$ about $\bar{p}$ such that $\cancel{\varphi}\, f|_V$ is a diffeo b/w V and $\cancel{\varphi}\, f(V)$.

f restricted to V.

Note:

Ex. $S_1 =$ xy-plane $\qquad S_2 =$ unit cylinder

$$\varphi(x,y,0) = (\cos x, \sin x, y)$$



"wraps" plane around cylinder

φ is not a global diffeo b/c not 1-1:

$$\varphi(x_0, y_0, 0) = \varphi(x_0 + 2\pi n, y_0, 0) \text{ for all } n.$$

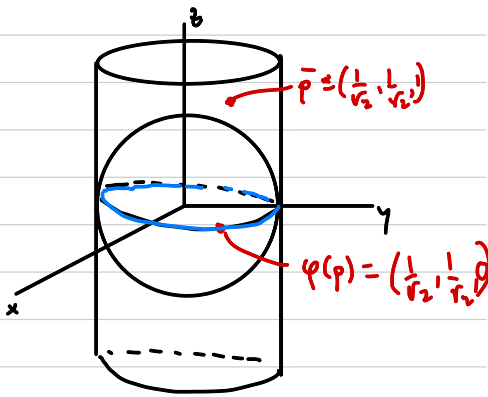
But at each $\bar{p} \in S$, can find a small open set V about $\bar{p}$

s.t. $\varphi : V \to \varphi(V)$ is 1-1

hence diffeo b/w these sets.

Thus φ is a local diffeo.

<u>Nonexample</u>



φ: cylinder $\to$ sphere.

$\varphi(x, y, z) = (x, y, 0)$

This map is not a local diffeomorphism.

Image of φ is equator in Sphere.

No way to choose open set V about $\bar{p}$ s.t. $\varphi : V \to \varphi(V)$ is 1-1.

$\hookrightarrow$ indicated by fact that $d\varphi_p$ not 1-1.