Isometries and Local Isometries

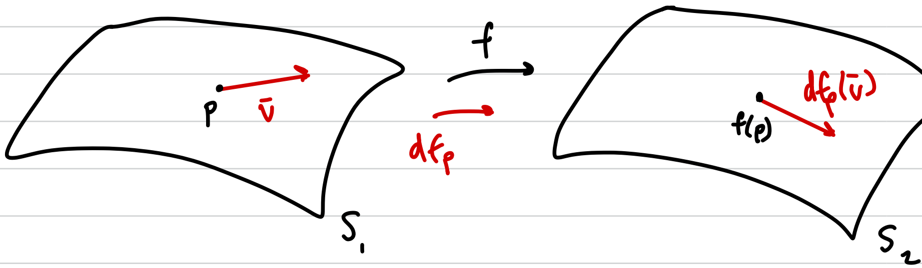
Recall: 3 levels of structure to surfaces:

- topological (open sets)
- differential (charts)
- geometric (first fundamental form/inner product)

A map $f: S_1 \longrightarrow S_2$ is a <u>diffeomorphism</u> if

- f is 1-1
- f is onto
- f is diff'ble
- f^{-1} is diff'ble

indicates that S_1 and S_2 are globally the same from point of view of differential structure.



Now that we have defined $df_p : T_pS_1 \to T_{f(p)}S_2$, we can consider the geometric structure.

<u>Defn</u> A diffeomorphism $f : S_1 \to S_2$ is an <u>isometry</u> if for all $\bar{p} \in S_1$ and pairs $\bar{v}, \bar{w} \in T_pS_1$,

$$(\bar{v} \cdot \bar{w})_p = (df_p(\bar{v}) \cdot df_p(\bar{w}))_{f(p)}$$

dot product eval at p

dot product eval at f(p).

(Equivalently : $I_p(\bar{v}) = I_{f(p)}(df_p(\bar{v}))$.)

In this case, we say S_1 and S_2 are <u>isometric</u>.

Defn indicates that S_1 and S_2 are globally same from point of view of geometric structure.

Ex If S_1 arrives at S_2 in $\mathbb{R}^3$ by a rigid motion

then S_1 and S_2 are isometric.

rotations and translations

Big idea: Since an isometry f preserves the first fundamental form it preserves all measurements defined via first fundamental form

e.g: length, angles, area, distance b/w pts.

we'll see: Gaussian curvature.