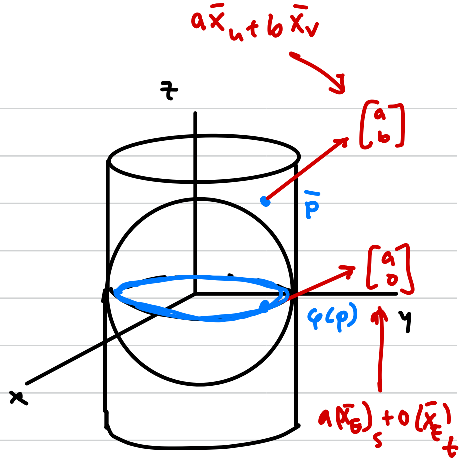
Ex. $\varphi : \overbrace{\text{cylinder}}^{S_1} \to \overbrace{\text{sphere}}^{S_2}$



$$\varphi(x, y, z) = (x, y, 0)$$

Let $\bar{p} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1\right)$

$\bar{x}(u,v) = (u, \sqrt{1-u^2}, v)$ $\qquad$ $\bar{x}_E(s,t) = (s, \sqrt{1-s^2-t^2}, t)$

Consider charts $(\bar{x}, U)$ and $(\bar{x}_E, V)$ about $\bar{p}$ and $\varphi(p)$ resp.

Recall from earlier: $\bar{x}_E^{-1} \circ \varphi \circ \bar{x}(u,v) = (\underbrace{u}_{\varphi_1(u,v)}, \underbrace{0}_{\varphi_2(u,v)})$

So $\quad d\varphi_p = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad \leftarrow$ * linear map $T_p S_1 \to T_{\varphi(p)} S_2$.

This makes sense: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix}$

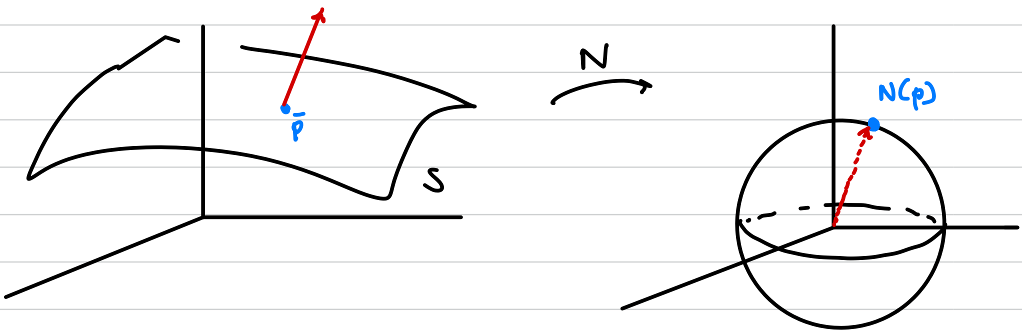
$d\varphi_p$ mimics vertical collapsing by φ.

Ex (Important : more to come)

Recall : the Gauss map

unit sphere

$$N : S \to S^2$$



dN_p indicated change of N

↳ will lead to curvature.