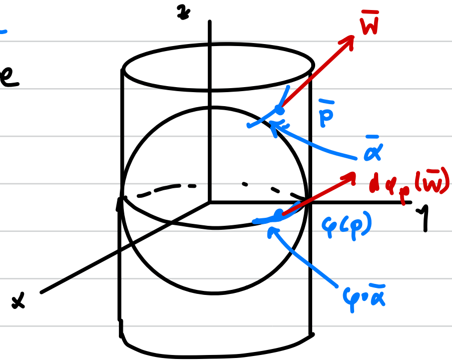
$$\underbrace{}_{}$$

Ex. Recall φ : $\overset{S_1}{\text{cylinder}} \to \overset{S_2}{\text{sphere}}$



$$\varphi(x, y, z) = (x, y, 0)$$

Let $\bar{p} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1\right) \in S_1$

so $\varphi(p) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \in S_2$.

Q: What does $d\varphi_p$ do?

Sps $\bar{w} \in T_p S_1$, with $\bar{w} = \bar{\alpha}'(0)$ for $\bar{\alpha}(t) = (x(t), y(t), z(t)) \in S_1$,

and $\bar{\alpha}(0) = \bar{p}$.

(So $\bar{w} = (x'(0), y'(0), z'(0))$.)

Then $(\varphi \circ \alpha)(t) = (x(t), y(t), 0)$

so $(\varphi \circ \alpha)'(\cancel{t}\ 0) = (x'(0), y'(0), 0)$

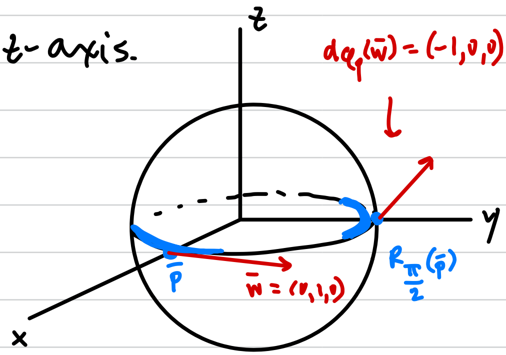
this is $d\varphi_p(\bar{w})$

Ex. Rotation of sphere.

Let S^2 denote the unit sphere in $\mathbb{R}^3$. Consider

$$R_{\frac{\pi}{2}} : S^2 \to S^2,$$

given by rotation $\frac{\pi}{2}$ about the z-axis.



Let $\bar{p} = (1,0,0)$.

Let $\bar{w} = \underbrace{(0,1,0)}_{\text{tangent vector, expressed as a vector in } \mathbb{R}^3} \in T_pS$

$\bar{\alpha}(t) = (\cos t, \sin t, 0)$ is a curve w/

$$\bar{\alpha}(0) = \bar{p} \qquad \text{and} \qquad \bar{\alpha}'(0) = \bar{w}$$

$(R_{\frac{\pi}{2}} \circ \bar{\alpha})(t)$
$(-\sin t, \cos t, 0)$
$(R_{\frac{\pi}{2}} \circ \alpha)'(t)$
$= (-\cos t, -\sin t, 0)$

Then $(R_{\frac{\pi}{2}} \circ \alpha)(0) = R_{\frac{\pi}{2}}(\bar{p}) = \underbrace{(0,1,0)}_{\text{image point of } R_{\frac{\pi}{2}}, \text{ in codomain } S^2}$

and $(R_{\frac{\pi}{2}} \circ \alpha)'(0) = \underbrace{(-1,0,0)}_{\text{tangent vector to } T_{R_{\frac{\pi}{2}}(p)} S^2} = d(R_{\frac{\pi}{2}})_p(\bar{w})$.