# Orientability and The Gauss Map

Recall: for a coordinate chart $(\bar{x}, U)$ on a regular surface $S$,

we have a unit normal vector to $T_pS$ at each

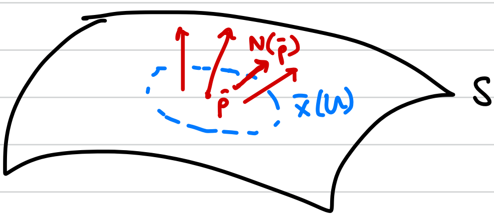
$\bar{p} \in \bar{x}(U)$ given by

$\{\bar{x}_u, \bar{x}_v\}$ a basis

$$N(p) = \frac{\bar{x}_u \times \bar{x}_v}{|\bar{x}_u \times \bar{x}_v|}$$

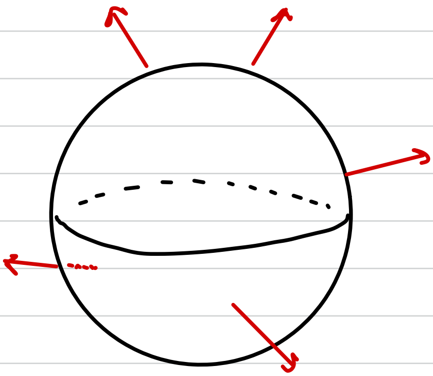
unit vector

By continuity, for a fixed coord. chart, all normal vectors defined this way point out the "same side" of $\bar{x}(U)$



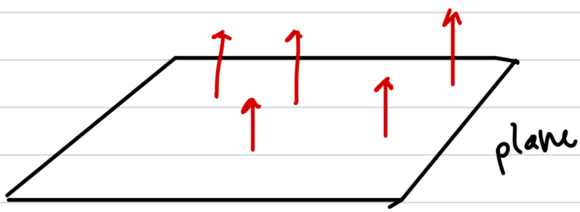
If it is possible to cover all of S <u>simultaneously</u> with charts so that $\bar{N}: S \to \mathbb{R}^3$ point out same side of S <u>always</u>, we say S is: <u>orientable</u>.

Ex.



sphere

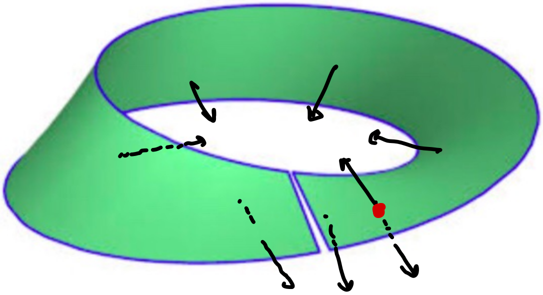
$\bar{N}$ out (or $\bar{N}$ in)



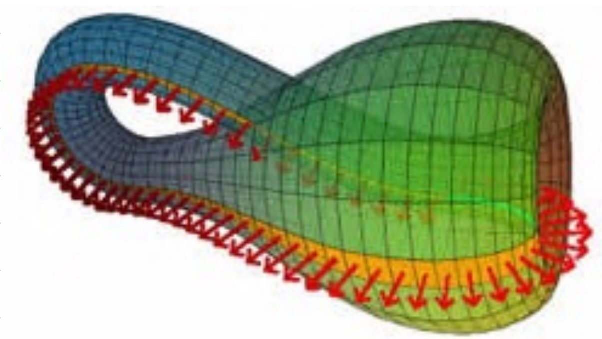

$\bar{N}$ up (or down)

Both are orientable surfaces.

Nonex.



Möbius band



Klein bottle