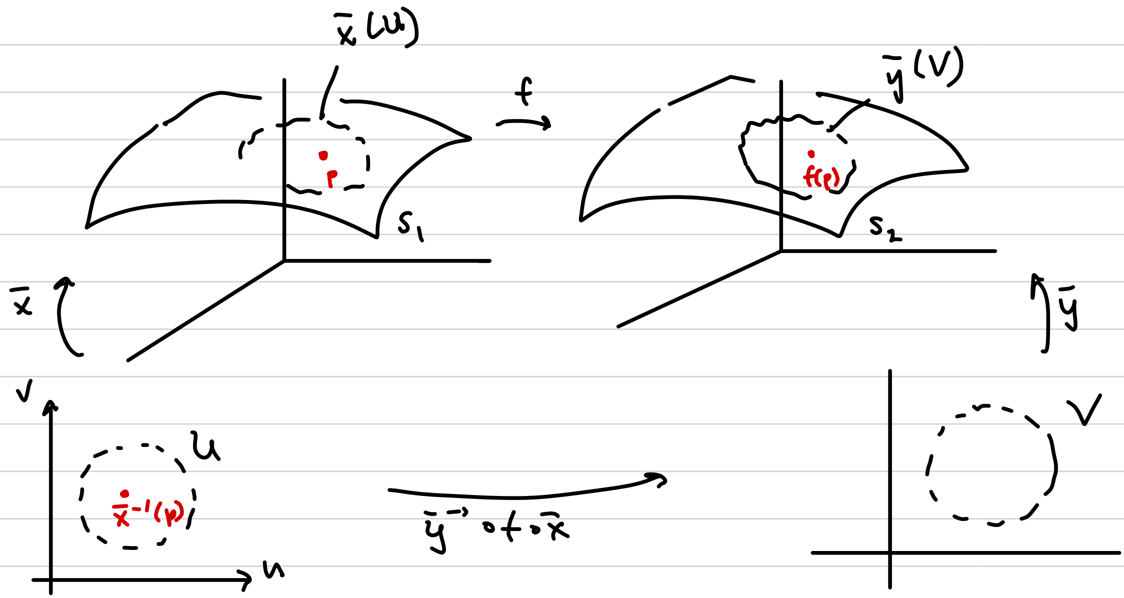
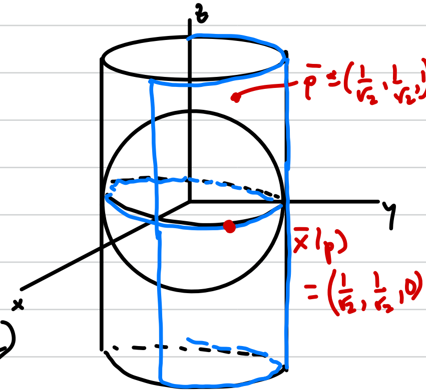
Defn Let S_1 and S_2 be regular surfaces. A map $f: S_1 \to S_2$ is differentiable at $\bar{p}$ if there exist charts $(\bar{x}, U)$ and $(\bar{y}, V)$ about $\bar{p}$ and $f(\bar{p})$ such that $\bar{y}^{-1} \circ f \circ \bar{x} : U \subset \mathbb{R}^2 \to V \subset \mathbb{R}^2$ is diff'ble at $\bar{x}^{-1}(p)$



* Can show defn is independent of choice of charts, similar to functions $f: S \to \mathbb{R}^n$.

Ex S_1 = unit cylinder about z-axis. S_2 = unit sphere



$$f : S_1 \longrightarrow S_2$$

$$f(x,y,z) = (x,y,0)$$

(Image of f is equator of sphere)

Chart about $\bar{p}$:

$(\bar{x}, U)$ where $U = \{(u,v) \mid -1 < u < 1\}$

$$\bar{x}(u,v) = (u, \sqrt{1-u^2}, v)$$

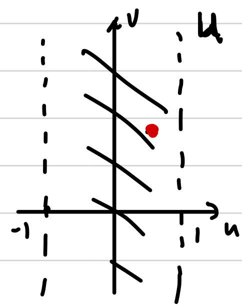
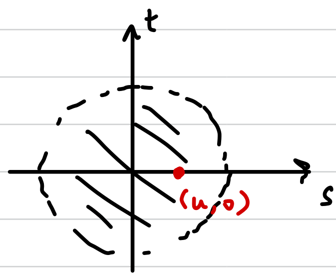


Chart about $f(\bar{p})$: $\bar{x}_E(s,t) = (s, \sqrt{1-s^2-t^2}, t)$

$(\bar{x}_E, V)$ where $V = \{(s,t) \mid s^2+t^2 < 1\}$



$$\bar{x}_E^{-1} \circ f \circ \bar{x}^{(u,v)} = x_E^{-1} \circ f\,(u, \sqrt{1-u^2}, v)$$

$$= \bar{x}_E^{-1}(u, \sqrt{1-u^2}, 0) \quad x_E^{-1} \circ f \circ x$$

$= (u,0)$ $\rightsquigarrow$ definitely diff'ble... so f diff'ble as map $S_1 \to S_2$.