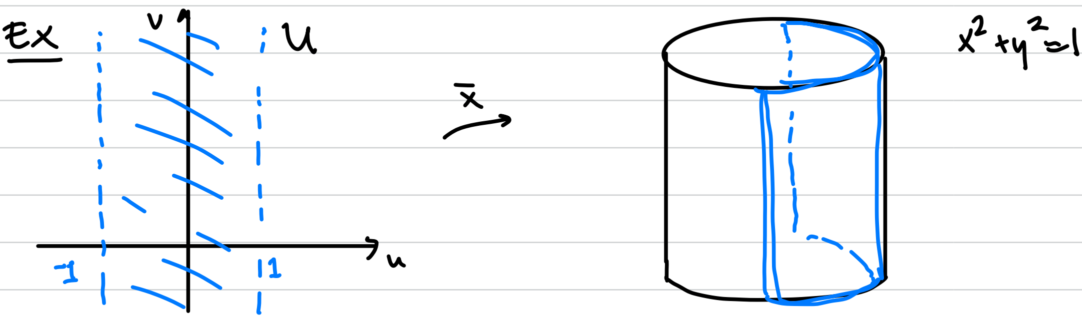
Defn If $F(u,v)=0$ (i.e. $\bar{x}_u \perp \bar{x}_v$) for all $(u,v)\in U$, we say $\bar{x}$ is an orthogonal parametrization.

If, in addition, $E(u,v)=G(u,v)$ for all $(u,v)\in U$, we say $\bar{x}$ is an isothermal parametrization.

Ex



$$\bar{x}(u,v) = (u, \sqrt{1-u^2}, v) \qquad -1<u<1,\ v\in\mathbb{R}$$

Then $\bar{x}_u = \begin{bmatrix} 1 \\ \frac{-u}{\sqrt{1-u^2}} \\ 0 \end{bmatrix}$ $\quad \bar{x}_v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

So $F(u,v)=0$ $\quad E(u,v) = 1+\frac{u^2}{1-u^2}$ $\quad G(u,v)=1$

Thus: $\bar{x}$ is an orthogonal but not isothermal param.

Some things we can measure using the first fundamental form:

1. For two curves $\bar{\alpha}$, $\bar{\beta}$ in S that both pass through a point $\bar{p} \in S$, we can measure the angle b/w the curves.

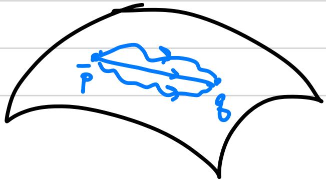
   ↳ given by angle b/w tangent vectors to curves at $\bar{p}$

2. For a curve $\bar{\alpha}$ in S, can measure the length of $\bar{\alpha}$ b/w $\bar{\alpha}(a)$ and $\bar{\alpha}(b)$

   ↳ given by $\int_a^b |\bar{\alpha}'(r)|\,dr = \int_a^b \sqrt{I_{\alpha(r)}(\alpha'(r))}\,dr$

3. More subtle: For two pts $\bar{p}$ and $\bar{q}$ in S, can measure the <u>distance</u> b/w $\bar{p}$ and $\bar{q}$:

$$d(\bar{p},\bar{q}) = \inf_{\substack{\text{curve } C \text{ from} \\ \bar{p} \text{ to } \bar{q}}} L(C)$$



So 1st Fund Form gives rise to a notion of <u>distance</u>.