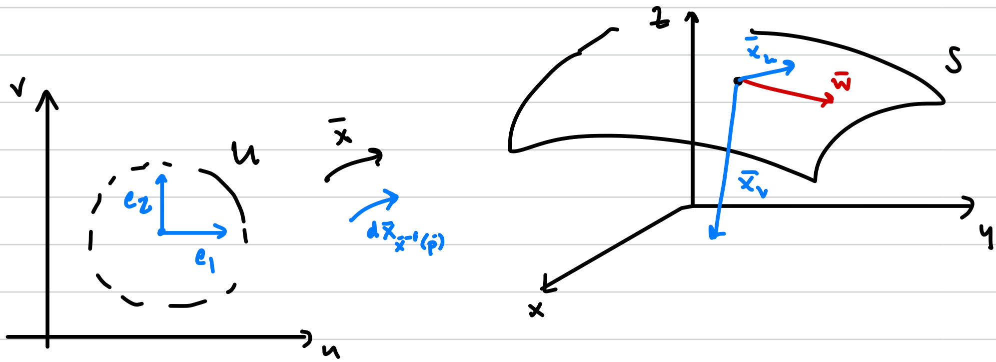
Local expressions of I_p



Sps $(\bar{x}, U)$ is a param. about $\bar{p} \in S$ so

$\{\bar{x}_u, \bar{x}_v\}$ is a basis for T_pS.

Sps $\bar{w} \in T_pS$ with $\bar{w} = c\bar{x}_u + d\bar{x}_v$ for some c, d.

Then
$$I_p(\bar{w}) = (c\bar{x}_u + d\bar{x}_v) \cdot (c\bar{x}_u + d\bar{x}_v)$$
$$= c^2(\bar{x}_u \cdot \bar{x}_u) + 2cd\,(\bar{x}_u \cdot \bar{x}_v) + d^2\,(\bar{x}_v \cdot \bar{x}_v)$$

So: For $\bar{p}$, if we know

$$\bar{x}_u \cdot \bar{x}_u$$

$$\bar{x}_u \cdot \bar{x}_v$$

$$\bar{x}_v \cdot \bar{x}_v$$

at $\bar{p}$, then we can find $I_p(\bar{w})$ for any $\bar{w} \in T_pS$.

Now, let $\bar{p}$ vary...

Given a chart $(\bar{x}, U)$, define functions $E, F, G : U \to \mathbb{R}$ by

each dot product eval. at $\bar{p}$

$$E(u_0, v_0) = \bar{x}_u \cdot \bar{x}_u$$

$$F(u_0, v_0) = \bar{x}_u \cdot \bar{x}_v$$

$$G(u_0, v_0) = \bar{x}_v \cdot \bar{x}_v$$

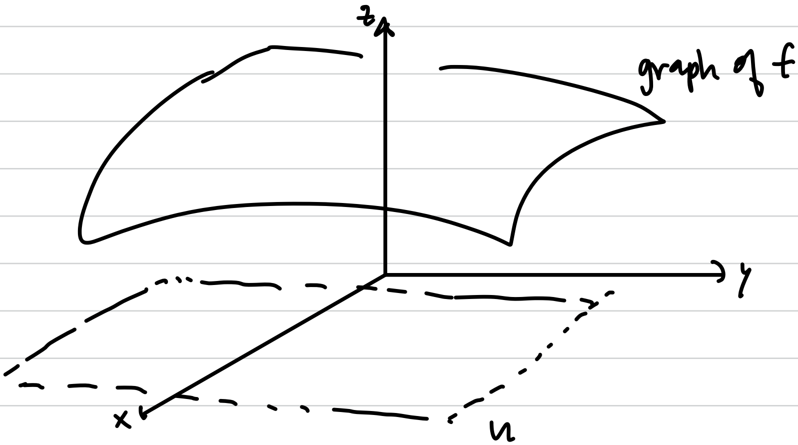
called: (local) component functions (or coefficients) of the first fundamental form.

where $\bar{p} = \bar{x}(u_0, v_0)$.

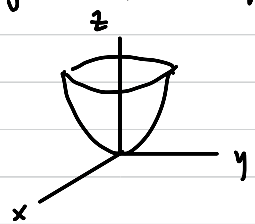
Letting (u_0, v_0) vary over U, we see how FFF varies from tangent plane to tangent plane.

regular surface.

EX. Sps. $f : U \subset \mathbb{R}^2 \to \mathbb{R}$. Let $S =$ graph of f.



e.g. $f(x,y) = x^2 + y^2$



Consider chart $\bar{x}(u,v) = (u, v, f(u,v))$.

$$\bar{x}_u = \begin{bmatrix} 1 \\ 0 \\ f_u(u,v) \end{bmatrix} \qquad \bar{x}_v = \begin{bmatrix} 0 \\ 1 \\ f_v(u,v) \end{bmatrix}$$

e.g. $\bar{x}_u = \begin{bmatrix} 1 \\ 0 \\ 2u \end{bmatrix}$ $\bar{x}_v = \begin{bmatrix} 0 \\ 1 \\ 2v \end{bmatrix}$

So $E(u,v) = 1 + (f_u(u,v))^2$ (e.g. $E = 1 + 4u^2$ $F = 4uv$ $G = 1 + 4v^2$)

$F(u,v) = f_u(u,v) f_v(u,v)$ } not 0, so in general $\bar{x}_u \not\perp \bar{x}_v$

$G(u,v) = 1 + (f_v(u,v))^2$