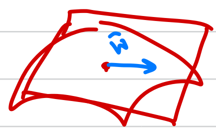
The First Fundamental Form

(An example of a metric... ** GEOMETRY!)

Defn Sps $S \subset \mathbb{R}^3$ is a regular surface. The <u>first fundamental form</u> on T_pS is the function



$$I_p : T_pS \longrightarrow \mathbb{R}$$

given by $I_p(\bar{w}) = \bar{w} \cdot \bar{w}$

dot product in $\mathbb{R}^3$.

Remarks

1. $I_p(\bar{w}) \geq 0$ for all $\bar{w} \in T_pS$. Equals 0 only when $\bar{w} = \bar{0}$. (positive-definiteness)

2. Knowing $I_p(\bar{w})$ for all $\bar{w} \in T_pS$ is equivalent to knowing $\bar{x} \cdot \bar{y}$ for all $\bar{x}, \bar{y} \in T_pS$.

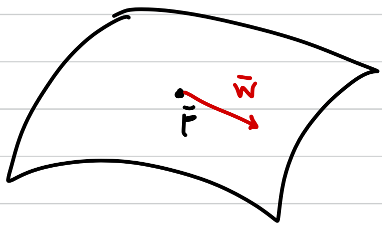
$$\begin{aligned} I_p(\bar{x}+\bar{y}) &= (\bar{x}+\bar{y}) \cdot (\bar{x}+\bar{y}) \\ &= \bar{x}\cdot\bar{x} + 2\bar{x}\cdot\bar{y} + \bar{y}\cdot\bar{y} \\ &= I_p(\bar{x}) + 2(x \cdot y) + I_p(\bar{y}) \end{aligned}$$

3. $I_{\bar p}(\bar w)$ is a quadratic form on T_pS

$a^2 q(\bar w)$

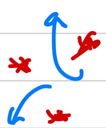
↳ a map $q: T_pS \to \mathbb{R}$ satisfying $q(a\bar w) = \cancel{a^2 \bar w}\ \forall \bar w.$

4. $\bar w$ is a 3-vector based at $\bar p \in S$



So $I_{\bar p}(\bar w) = \bar w \cdot \bar w$ means dot product in $\mathbb{R}^3$.

We are restricting the dot product in $\mathbb{R}^3$ (based at $\bar p$) to T_pS.



GEOMETRY!

5. By introducing discussion of I_p (equiv to dot product), we can discuss lengths, angles of vectors in T_pS.

We can extend this to measurements of angles, lengths, area, curvature (ie. geometry) on surfaces.