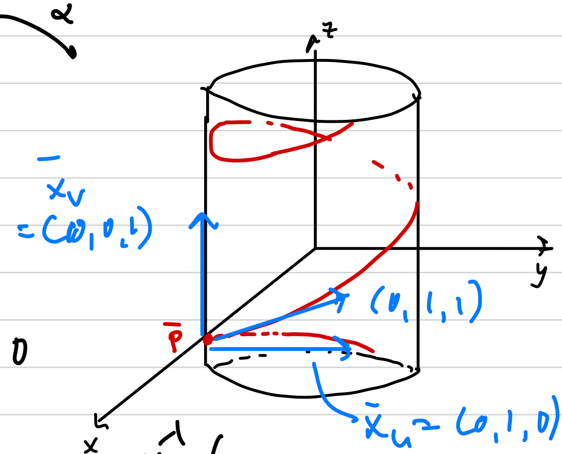
Ex. S = unit cylinder $x^2+y^2=1$ in $\mathbb{R}^3$.

Sps. $\bar{\alpha}(t) = (\cos t, \sin t, t)$, $t \in \mathbb{R}$.

helix on cylinder.



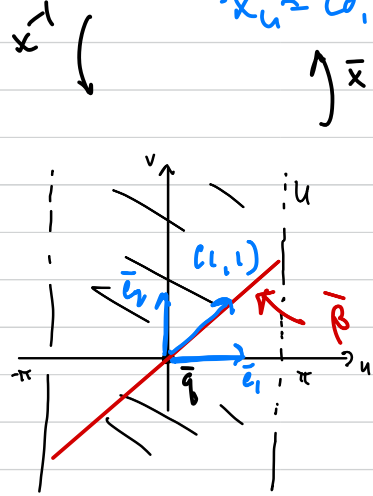
$\bar{\alpha}(0) = \bar{p} = (1,0,0)$

$\bar{w} = \bar{\alpha}'(0) = (-\sin t, \cos t, 1)$ at 0

$= (0,1,1)$.

Consider chart $\bar{x}(u,v) = (\cos u, \sin u, v)$

$-\pi < u < \pi$, $v \in \mathbb{R}$.



$\bar{q} = \bar{x}^{-1}(\bar{p}) = (0,0)$

Note: $(\bar{x}_u)_{\bar{p}} = (-\sin 0, \cos 0, 0)$

$= (0,1,0)$

$(\bar{x}_v)_{\bar{p}} = (0,0,1)$

$$\bar\beta(t) = (\bar x^{-1}\circ\bar\alpha)(t) = (\underbrace{t}_{u(t)}, \underbrace{t}_{v(t)})$$

Discussion about local basis says:

$$\bar w = u'(0)\,\bar x_u + v'(0)\,\bar x_v$$

$$= (1)\,\bar x_u + (1)\,\bar x_v$$

$$= \begin{bmatrix}1\\1\end{bmatrix}$$ in basis $\{\bar x_u, \bar x_v\}$ of T_pS.

(as a vector in ambient space $\mathbb{R}^3$, relative to std. basis of $\mathbb{R}^3$

$$\bar w = \begin{bmatrix}0\\1\\1\end{bmatrix}$$)

look the same

vector based at $\bar q$

Note: $\bar\beta'(0) = 1\bar e_1 + 1 e_2 = \begin{bmatrix}1\\1\end{bmatrix}$

Idea: $\bar w$ has same relationship to $\{\bar x_u, \bar x_v\}$ in T_pS that $\bar\beta'(0)$ has to $\{\bar e_1, \bar e_2\}$ in tangent space to U at $\bar q$ (b/c $d\bar x_{\bar q}$ is a linear transformation.