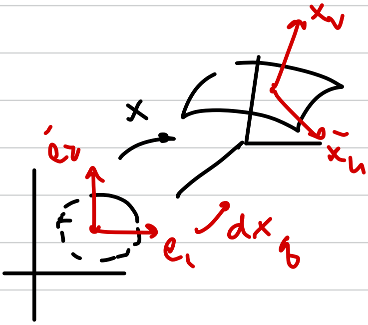
The Basis $\{\bar{x}_u, \bar{x}_v\}$ of T_pS.

Recall: if $\bar{x}(u,v) = (x(u,v),\ y(u,v),\ z(u,v))$

is a smooth map (e.g. a parametrization)

then

$$\bar{x}_u = d\bar{x}_q(\bar{e}_1) = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\bar{e}_1} = \begin{bmatrix} \frac{\partial x}{\partial u} \\ \frac{\partial y}{\partial u} \\ \frac{\partial z}{\partial z} \end{bmatrix}$$

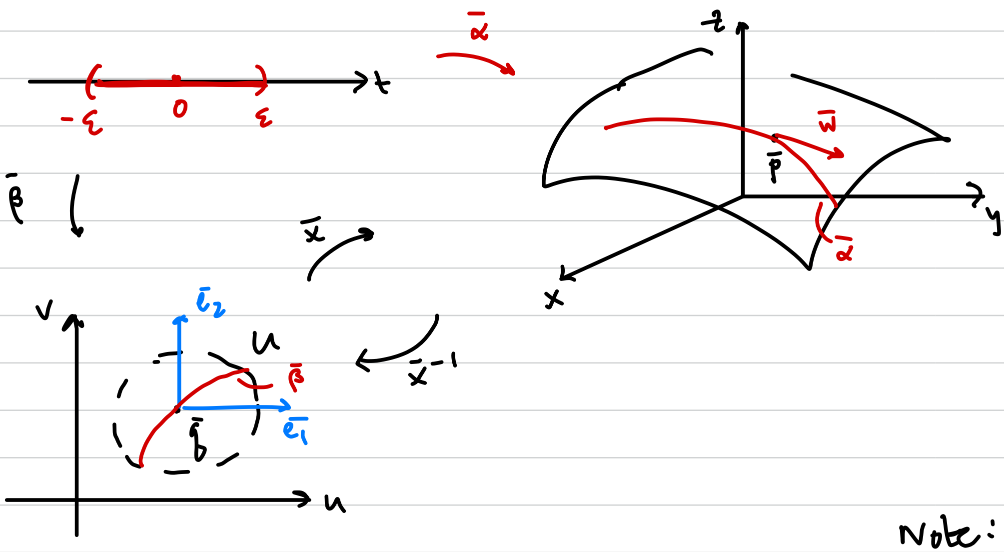


vectors in $\mathbb{R}^3$, based at $\bar{p}$.

$$\text{and } \bar{x}_v = d\bar{x}_q(\bar{e}_2) = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{bmatrix} \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\bar{e}_2} = \begin{bmatrix} \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial v} \end{bmatrix}$$

Sps $\bar{w} \in T_pS$, so $\bar{w} = c\bar{x}_u + d\bar{x}_v$ for some $c, d \in \mathbb{R}$.

GOAL: Find an expression for c and d.

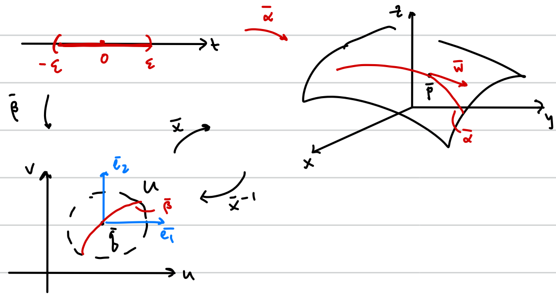


Note: $\bar{\beta}(0) = \bar{q}$.

$\bar{w} = \bar{\alpha}'(0) = (\bar{x} \circ \bar{\beta})'(0)$ where $\bar{\beta} = \bar{x}^{-1} \circ \bar{\alpha}$

So: $\bar{\beta}$ a curve in $\mathbb{R}^2$. Sps $\bar{\beta}(t) = (\underset{\sim}{u(t)}, \underset{\sim}{v(t)})$.

component functions



Then: $\bar{w} = \bar{\alpha}'(0)$

$$= (\bar{x} \circ \bar{\beta})'(0)$$

reexpress

$$= d(\bar{x} \circ \bar{\beta})_0 (\bar{e}_1)$$

chain rule

$$= d\bar{x}_{\bar{q}}\, d\bar{\beta}_0 (\bar{e}_1)$$

reexpress

$$= d\bar{x}_q (\beta'(0)).$$

standard matrices

$$= \begin{bmatrix} | & | \\ \bar{x}_u & \bar{x}_v \\ | & | \end{bmatrix} \begin{bmatrix} u'(0) \\ v'(0) \end{bmatrix}$$

#'s.

$\bar{\beta}'(0)$ in $\mathbb{R}^2$

$$= \underbrace{u'(0)}_{c}\, \bar{x}_u + \underbrace{v'(0)}_{d}\, \bar{x}_v .$$

vector in $T_p S$ .... in $\mathbb{R}^3$.

$A\bar{x}$

$$\begin{bmatrix} | & | & & | \\ \bar{a}_1 & \bar{a}_2 & \cdots & \bar{a}_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$= x_1 \bar{a}_1 + x_2 \bar{a}_2 + \ldots + x_n \bar{a}_n$

So, in terms of basis $\{\bar{x}_u, \bar{x}_v\}$ of $T_p S$, $\bar{w} = \begin{bmatrix} u'(0) \\ v'(0) \end{bmatrix}$.

CAREFUL:

!! <u>Not</u> same as $\bar{\beta}'(0)$!! ⇝ live in diff. spots.