Remarks:

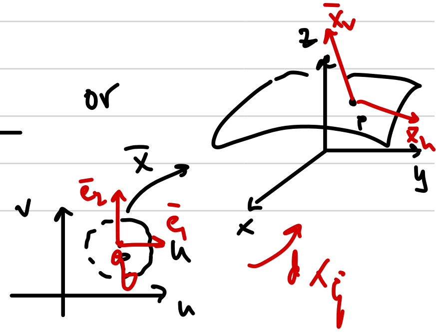
1. Since T_pS is defined to be the set of tangents to curves in S (and since the definition of tangent vectors does not depend on a choice of parametrization), if $(\bar{x}, U)$ and $(\bar{y}, V)$ are two parametrizations about $\bar{p}$,

$$d\bar{x}_{\bar{x}^{-1}(p)}(\mathbb{R}^2) = d\bar{y}_{\bar{y}^{-1}(p)}(\mathbb{R}^2)$$

both equal to T_pS.

2. Given a parametrization $(\bar{x}, U)$ about $\bar{p}$, we get a basis $\{\bar{x}_u, \bar{x}_v\}$ (a.k.a. $\left\{\frac{\partial \bar{x}}{\partial u}, \frac{\partial \bar{x}}{\partial v}\right\}$, a.k.a. $\{d\bar{x}_q(\bar{e}_1), d\bar{x}_q(\bar{e}_2)\}$) of T_pS.

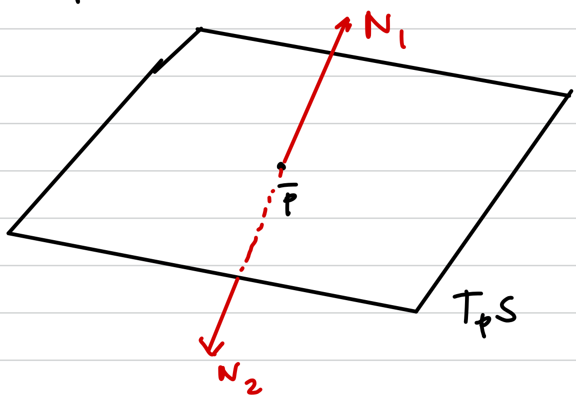
This is called the <u>local basis from $\bar{x}$</u> or <u>basis associated to $\bar{x}$</u>.



Note: even though $\bar{e}_1 \perp \bar{e}_2$ in $T_{\bar{x}^{-1}(p)} U$,

it may be that $\bar{x}_u \not\perp \bar{x}_v$ in $T_p S$.

3. In $\mathbb{R}^3$, $T_p S$ has 2 unit normals:



Given a param. $(\bar{x}, U)$, we can uniquely define N by setting

$$N = \frac{\bar{x}_u \times \bar{x}_v}{|\bar{x}_u \times \bar{x}_v|}$$

(here, order of $\{\bar{x}_u, \bar{x}_v\}$ matters)