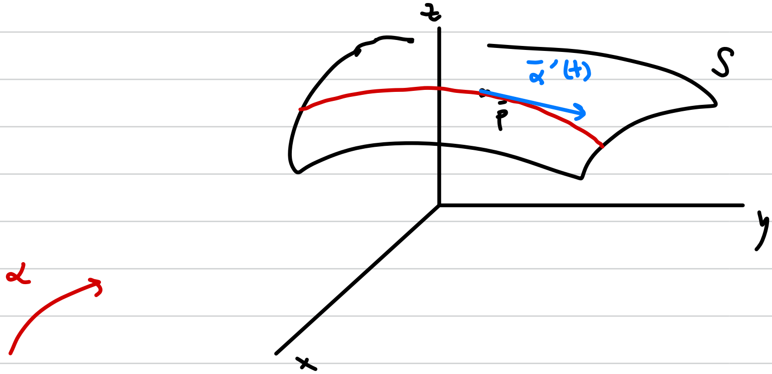
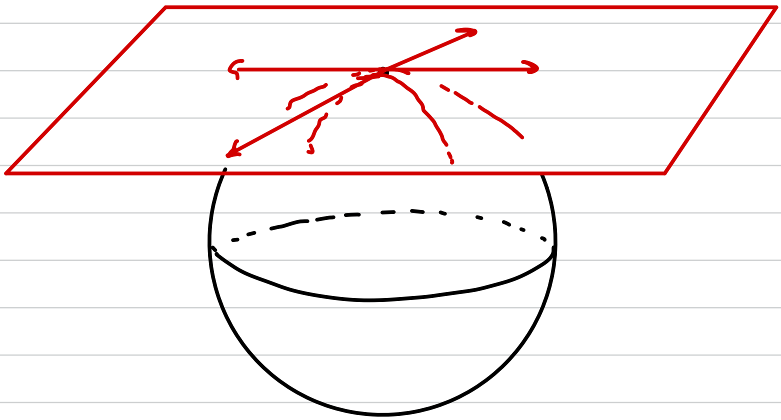
The Tangent Plane to S at $\bar{p}$

<u>Defn</u> Sps. $S \subset \mathbb{R}^3$ is a regular surface and $\alpha : (-\varepsilon, \varepsilon) \to S$ is a curve whose trace lies in S. Sps. $\bar{\alpha}(0) = \bar{p}$.

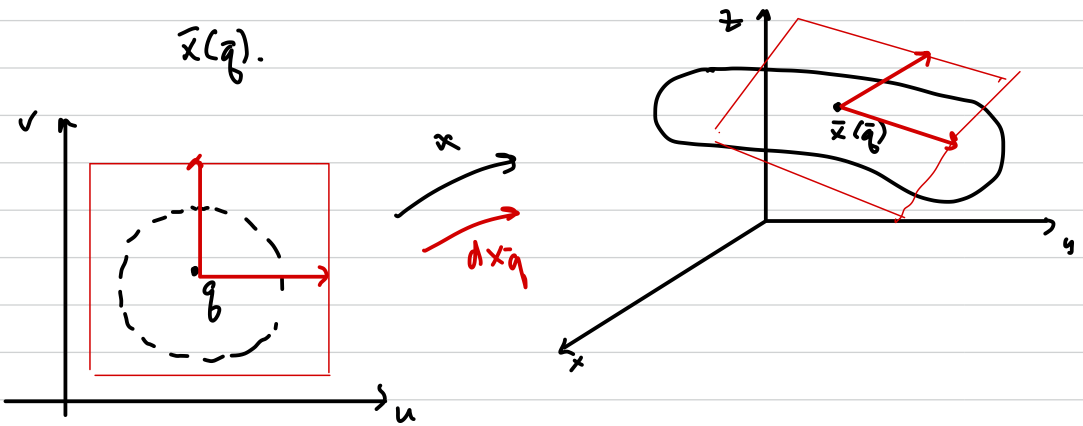


Then $\bar{\alpha}'(0)$ is a <u>tangent vector to S at $\bar{p}$</u>.

The set of all tangent vectors is the <u>tangent plane to S at p</u>, denoted T_pS.



Recall: for a map $\bar{x} : \mathbb{R}^2 \to \mathbb{R}^3$, $d\bar{x}_q$ is a linear approximation of $\bar{x}$ near $\bar{q}$. It linearly maps vectors based at $\bar{x}$ to vectors based at $\bar{x}(\bar{q})$.



Prop. Let $\bar{x} : U \subset \mathbb{R}^2 \to S$ be a parametrization such that $\bar{p} = \bar{x}(\bar{q})$ where $\bar{q} \in U$. Then

$$d\bar{x}_{\bar{q}}(\mathbb{R}^2) = T_p S$$

vectors in $\mathbb{R}^2$, based at $\bar{q}$

tangent vectors to curves in S through $\bar{p}$.

Idea: • $d\bar{x}_{\bar{q}}$ has rank 2 so $\dim d\bar{x}_{\bar{q}}(\mathbb{R}^2) = 2$.

- Because $d\bar{x}_{\bar{q}}$ is linear, $d\bar{x}_q(\mathbb{R}^2)$ is a subspace of $\mathbb{R}^3$.

- GOAL: show it coincides with $T_p S$.