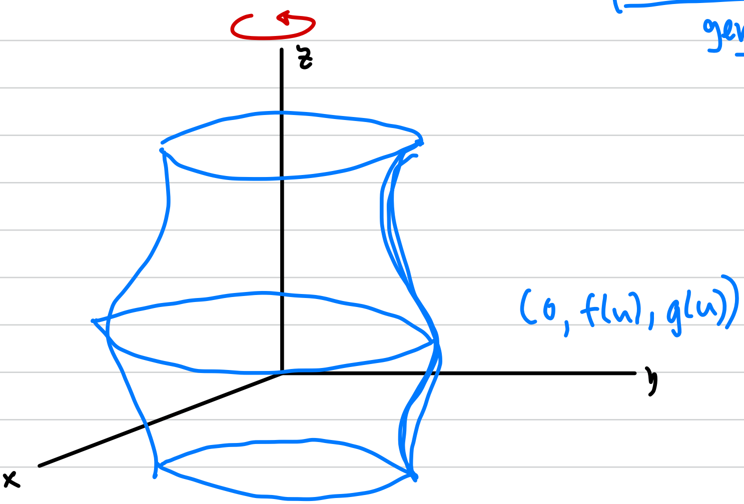
Ex Surfaces of Revolution

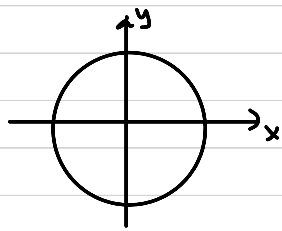
Sps. $\bar{\alpha}(u) = (0, f(u), g(u))$, $u \in I$ is a regular curve w/o self-intersection.

↳ Sps $f(u) > 0$ for all u.

Note that $\bar{\alpha}(u)$ lies in: yz-plane.
and is called a <u>profile curve</u> or <u>generating curve</u>



To spin $\bar{\alpha}$ around z-axis, consider

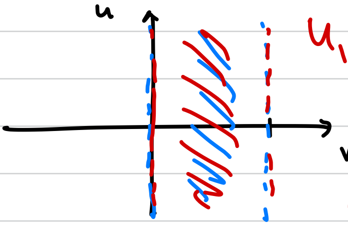


gives radius of circle.

$$\bar{x}(u,v) = (\sin v\, f(u), \cos v\, f(u), g(u))$$

give circular nature

What should ranges of u and v be to make this a coordinate chart?

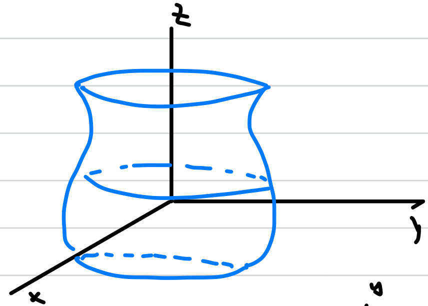


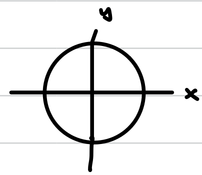
$(\bar{x}_1, U_1) \longrightarrow$

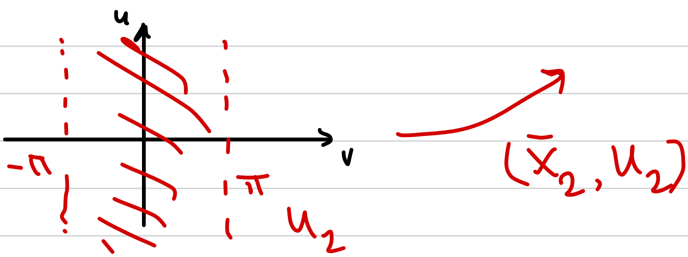
$u \in I$

$0 < v < 2\pi$

* U open







Here, for fixed choice of u_0, curves $\bar{x}(u_0, v)$ (v varies) are called a <u>parallel</u>.

And for fixed choice of v_0, curves $\bar{x}(u, v_0)$ are called a <u>meridian</u>.

Check that $\bar{x}(u,v) = (\sin v\, f(u), \cos v\, f(u), g(u))$, $u \in I$, $0 < v < 2\pi$

is a chart:

1. $\bar{x}$ diff'ble? ✓ yes b/c $\cos v, \sin v, f(u), g(u)$ all have ~~partials~~ derivatives of all order.

2. $\bar{x}$ cts? $\bar{x}$ 1-1? $\bar{x}^{-1}$ cts? *

↳ yes. all component fncs. cts.

3. $$d\bar{x}_q = \begin{bmatrix} \sin v\, f'(u) & \cos v\, f(u) \\ \cos v\, f'(u) & -\sin v\, f(u) \\ g'(u) & 0 \end{bmatrix}$$ ... has rank 2. ✓

$$\begin{bmatrix} \sin v \\ \cos v \end{bmatrix} \cdot \begin{bmatrix} \cos v \\ -\sin v \end{bmatrix} = 0$$

↳ $\perp$ ... not parallel