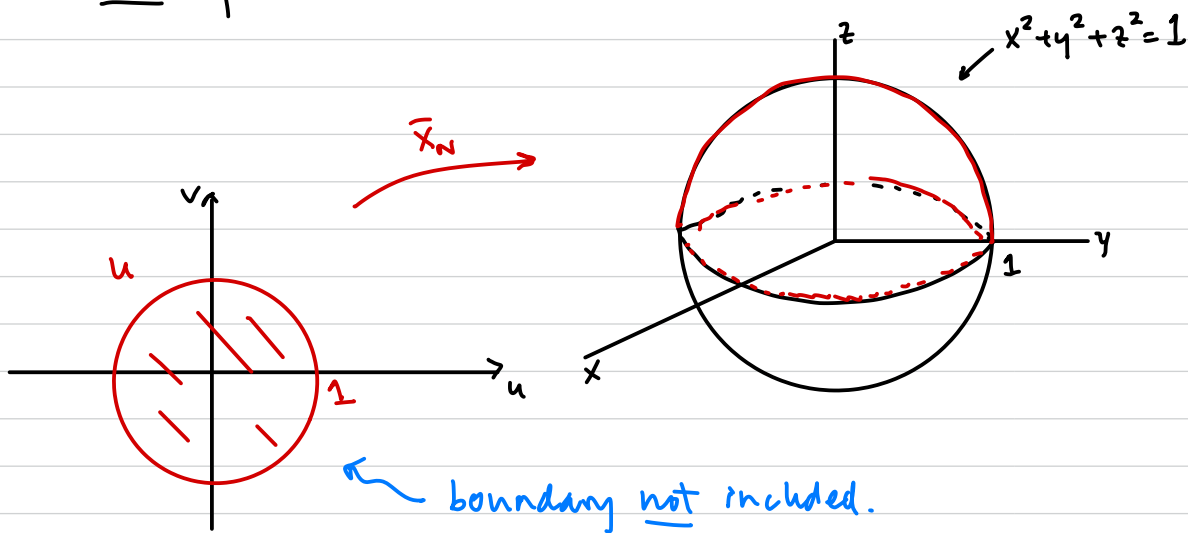


(unit)
Ex. Sphere.



$$U = \{(u, v) \mid u^2 + v^2 < 1\}$$

Northern hemisphere chart:

$$\bar{x}_N(u, v) = (u, v, \sqrt{1 - u^2 - v^2}), \quad u^2 + v^2 < 1.$$

$$\bar{x}_N(u, v) = (u, v, \sqrt{1-u^2-v^2}), \quad u^2 + v^2 < 1$$

Check: 1. diffeable? quick check: check partials. -

do they exist,
all orders?

$$x(u, v) = u \quad \checkmark$$

$$y(u, v) = v \quad \checkmark$$

$$z(u, v) = \sqrt{1-u^2-v^2}$$

\int ok w/ b/c $u^2 + v^2 < 1$.

$$\hookrightarrow \frac{\partial z}{\partial u} = \frac{-u}{\sqrt{1-u^2-v^2}}$$

$$\frac{\partial z}{\partial v} = \frac{-v}{\sqrt{1-u^2-v^2}} \quad \checkmark$$

2. cts? \checkmark

1-1? \checkmark

inverse: $\bar{x}_N^{-1}(x, y, z) = (x, y)$. (cts)

$$3. \bar{p} = (u, v) \quad d\bar{x}_N(u, v) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{-u}{\sqrt{1-u^2-v^2}} & \frac{-v}{\sqrt{1-u^2-v^2}} \end{bmatrix}$$

rank? 2. \checkmark

Also need other charts: $\bar{x}_S, \bar{x}_E, \bar{x}_W, \bar{x}_F, \bar{x}_B$.

Q: smallest # of charts needed to cover?