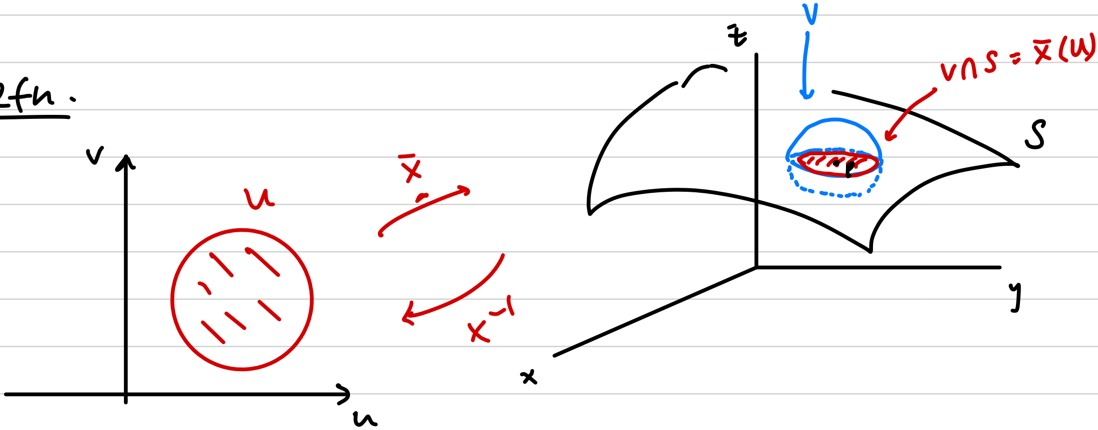
We are now ready for the defn of a regular surface:

Defn.



A set $S \subset \mathbb{R}^3$ is a <u>regular surface</u> if for each $\bar{p} \in S$ there exists a neighborhood (i.e. open ball) $V \subset \mathbb{R}^3$ containing $\bar{p}$ and a map $\bar{x} : U \to V \cap S$ of an open set $U \subset \mathbb{R}^2$ onto $V \cap S$ such that:

1. $\bar{x}(u,v) = (x(u,v), y(u,v), z(u,v))$ is diff'ble.

   ↳ quick check: partials all exist and are cts.

   allows us to define $\bar{x}^{-1}$.

2. $\bar{x}$ is continuous, 1-1, and $\bar{x}^{-1}\big|_{V \cap S}$ is continuous,

   ↳ quick check: $\bar{x}$ is cts. if all component fncs are cts.

and....

3. For each $\bar{q} \in U$, the derivative $d\bar{x}_{\bar{q}} : \mathbb{R}^2 \to \mathbb{R}^3$ is one-to-one. Equivalently, the Jacobian matrix $d x_{\bar{q}}(u,v) = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{bmatrix}$ has rank: 2.

Note: for each pt $\bar{p} \in S$ there must be such a map $\bar{x}$.

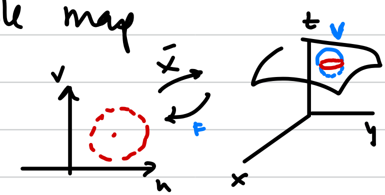
$(\bar{x}, U)$ is called a chart or parametrization or a coordinate neighborhood.

Note: An implication of this definition is that for every set V as above, there is a diff'ble map

in $\mathbb{R}^3$

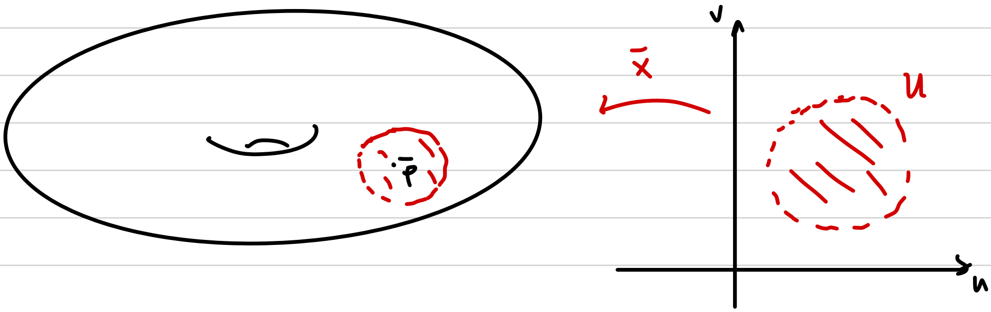
in $\mathbb{R}^3$

$$F : V \to \mathbb{R}^2$$



such that $\bar{F} \circ \bar{x}(u,v) = F(\bar{x}(u,v)) = (u,v) \ \forall u,v \in U$.

Idea:



When you zoom in to a regular surface, every point is contained in a region that "looks like" $\mathbb{R}^2$

made rigorous by defn.