Handy fact:

Sps $f: \mathbb{R}^2 \to \mathbb{R}^3$, $f(u,v) = (x(u,v), y(u,v), z(u,v))$.

If each component function x, y, z has cts partial derivatives of all orders at $\bar{p}$, then f diff'ble at $\bar{p}$.

note: NOT defn of diff'bility. These conditions imply diff'bility.

Sps. $f: \mathbb{R}^2 \to \mathbb{R}^3$ is diff'ble at $\bar{p}$.

a.k.a. Jacobian

Q: what is standard matrix of $df_{\bar{p}}$ at $\bar{p}$?

$$df_{\bar{p}} = \begin{bmatrix} \frac{\partial x}{\partial u}(\bar{p}) & \frac{\partial x}{\partial v}(\bar{p}) \\ \frac{\partial y}{\partial u}(\bar{p}) & \frac{\partial y}{\partial v}(\bar{p}) \\ \frac{\partial z}{\partial u}(\bar{p}) & \frac{\partial z}{\partial v}(\bar{p}) \end{bmatrix}$$

called f_u and f_v

$\underline{\text{Ex}}$ $f(u,v) = (u^2 v,\ ue^v,\ u\cos v)$

$$df_{(u,v)} = \begin{bmatrix} 2uv & u^2 \\ e^v & ue^v \\ \cos v & -u\sin v \end{bmatrix}$$

"Recall": the chain rule.

Sps $f : \mathbb{R}^n \to \mathbb{R}^p$ is diffble at $\bar{p}$ and

$g : \mathbb{R}^p \to \mathbb{R}^m$ is diffble at $f(\bar{p})$. Then

$g \circ f : \mathbb{R}^n \to \mathbb{R}^m$ is diffble at $\bar{p}$

and

$$d(g \circ f)_p = dg_{f(\bar{p})}\, df_p .$$